A century of retaining wall computation methods

III: Modeling of retaining walls by means of the finite element method

Luc DELATTRE
Laboratoire Central des Ponts et Chaussées

INTRODUCTION

The finite element method is a very general tool that enables identifying a numerical solution to any problem that can be described using a set of partial derivative equations over a finite domain. Various method extensions moreover serve to take account of spatial or temporal discontinuities in the targeted solutions, infinite extension domains, etc. This method offers the possibility of incorporating of highly-sophisticated behavioral models, which are more realistic than those included within more conventional methods, and it is only normal therefore that the finite element method has incited considerable interest on the part of both geotechnical engineers and civil engineers.

After the initial applications to dams [Clough and Woodward, 1967], excavations [Chang and Duncan, 1970] and gravity walls [Duncan and Clough, 1971; Clough and Duncan, 1971], use of the finite element method for computing shest piles and digphragm walls began during the 1970's [Bjerrum *et al.*, 1972; Egger, 1972; Clough *et al.*, 1972; Palmer and Kenney, 1972]. These first analyses were followed by major efforts for examining in a systematic fashion the potential contributions of the method in computing geotechnical structures in general and retaining structures in particular [Duncan, 1994; Gens, 1995].

The use of finite elements in geotechnical engineering nonetheless remains somewhat limited in engineering practice, with the remarkable exception of tunnels, due to the method's effectiveness when combined with the convergence-confinement method for analyzing, within a two-dimensional configuration, the entire tunnel excavation process, which by nature is three-dimensional.

One of the explanations behind the method's limited practical applications is that the more conventional computation methods have for a long time been yielding satisfactory results, at least for purposes of verifying structural design with respect to a structure's ultimate limit states. In contrast, these methods typically yield little information on the displacements and deformations of structures, which may become detrimental in certain contexts (in an urban area, for example, it is useful to be able to predict the impact on adjacent buildings when performing an excavation intended to accommodate an underground parking lot; in this case, conventional computation methods prove unsuitable).

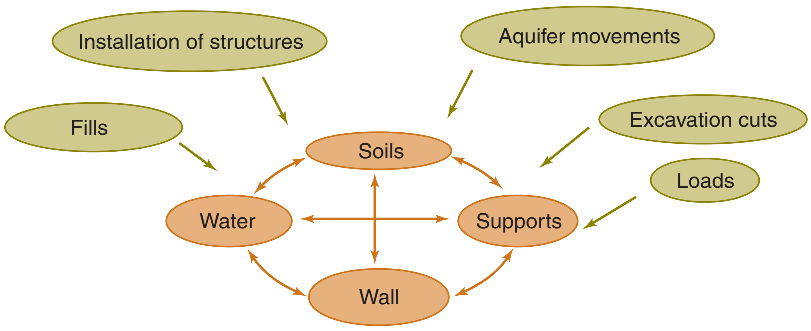
Another reason pertains to the specificities of geotechnical problems, which serve to raise the cost of the modeling exercise: in most instances, structures entail the use of natural materials whose behavior is more difficult to identify than that of industrialized artificial materials, and moreover the targeted system is evolving over time. The entire construction process presupposes in essence the provision and removal of materials, which corresponds to a sequence of often-complex steps.

In the special case of excavations and retaining structures, the finite element method has for the past thirty years been considered as the designated substitute for more conventional computation methods, given their specific limitations. However, the specificities of the geotechnical problems cited above serve as complicating factors for method implementation and have limited the extent of its application.

Conventional computation methods present the major disadvantage of only processing in a viable way a set of very specific structural forms, for which unique solutions have been derived [Delattre, 2002; Delattre and Marten, 2003]. Cited below are just the most obvious of such usage restrictions:

- ground geometry can only be incorporated in a rigorous manner within a few configurations; hence, active and passive stresses are strictly expressed for structural configurations in which: the natural terrain has been limited by a plane, whether horizontal or inclined; the interfaces between soil layers are parallel to the ground; and the structure has undergone a special type of kinematic pattern;
- interactions of the wall with other structural components can only be taken into account by a stress torsor applicable to the wall, which does not enable recognizing forms of complex interaction, such as those due to neighboring structures; modeling a retaining wall in the vicinity of a foundation, tunnel or other retaining wall thus proves inaccessible for conventional methods. Similarly, the interaction of the retaining wall with other structural components, e.g. wing walls, remains an intricate exercise;
- modeling the soil-structure interaction relies upon notions for which no consensus has yet to be reached, e.g. the reaction coefficient.

Confronted with these limitations, the approach proposed by the finite element method is intended to explicitly include the behavior of each structural part and the associated interactions, for the various types of loadings, so as to reproduce the entire set of structural behavior features (see Fig. 1).Without reviewing the underlying theoretical aspects in detail, the following discussion will present the various structural behavior features that the finite element method is now able to access.



■ ***Fig. 1***
Relations involved in the modeling of retaining walls

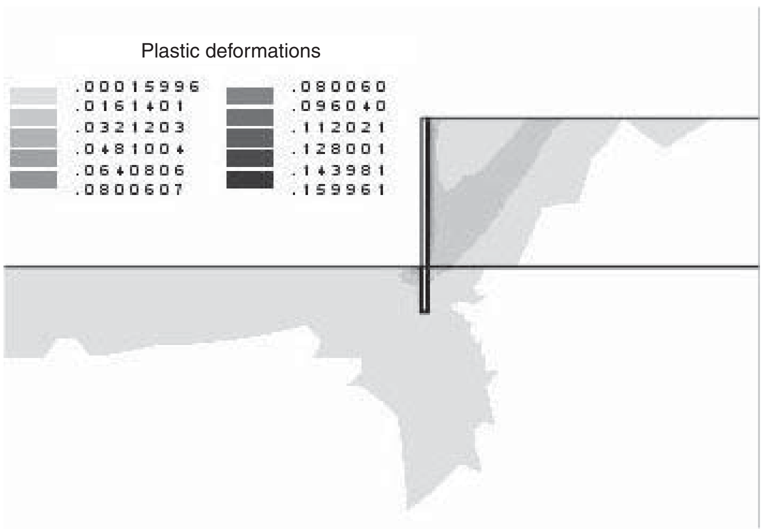
A FEW ASPECTS INHERENT IN THE MODELING OF RETAINING STRUCTURES BY USE OF FINITE ELEMENTS

The finite element method, above all, seeks an approximated solution to a problem defined by a system of partial derivative equations. The physics represented by this problem obviously depends not only on the options available within a given software application, but even more so on the choices made by the finite element modeler. The primary ingredients of the numerical modeling approach that need to be selected will be briefly discussed herein, along with the advantages and disadvantages of the different programming options.

Behavioral modeling of the various structural elements

Soil

As opposed to the vast majority of other civil engineering structures, retaining structures are characterized by the fact that during their service life, sizable soil zones are submitted to a plasticity regime. Such is the case for supported grounds, which very often lie at the active limit state of equilibrium; this would also apply to cases of soils located in foundation pits, which might enter the plasticity domain either as a result of the unloading experienced during excavation or by virtue of lateral compression due to the retaining structure (see Fig. 2).



■ ***Fig. 2***
Zones in plasticity in the vicinity of an excavation support: The case of a braced wall [Delattre, 1999]

The presence of plastic zones serves to limit the use of behavioral laws that do not provide the possibility for determining the state of soil failure. The field of application of linear elasticity is thus restricted to the short-term analysis of excavations within over-consolidated stiff clays, which as a result of their high strength are not lead to failure. Cole and Burland [1972] and then Ward and Burland [1973, in Burland *et al.*, 1979] made use of such linear elasticity to evaluate, under undrained conditions, the deformations due to excavations conducted in over-consolidated clays throughout the London metropolitan basin.

The association of linear elasticity with a Mohr-Coulomb rupture criterion composed the simplest elastoplastic model and the one most commonly employed for modeling soil behavior in both retaining wall and excavation problems. The behavioral law parameters are limited in number and accessible by means of widespread tests: shear strength determined using triaxial tests in the laboratory, and elasticity parameters from either triaxial tests or pressuremeter tests [Dauvisis and Menard, 1964].

The numerical simulations that rely upon this behavioral law yield however an imperfect assessment of the behavior of retaining walls and excavations. The execution of retaining walls and excavations actually incites a significant loading on those soil zones located near the structure, with this loading dropping as distance to the structure increases. Such a variation has been observed for deformations imposed on soil adjacent to the structure (see Fig. 3). The application of behavioral laws that rely upon an independent elasticity modulus of the deformation does not enable deriving accurate deformation values at every point of the model, but merely generates average model deformation. In particular, it may be noted that this approach leads to propagating deformations at greater distances from the structure.

The rate of material confinement also plays a significant role and results in an increase in material stiffness as the average stress being applied increases. For granular materials therefore, the elasticity analysis of the behavior of a particle assembly shows that the volumic compression modulus is proportional to a 1/3 to 1/2 power of the average applied stress [Mestat, 1993], whereas a proportionality between average stress and elasticity modulus proves more representative of the behavior of stiff clays [Jardine *et al.*, 1991].

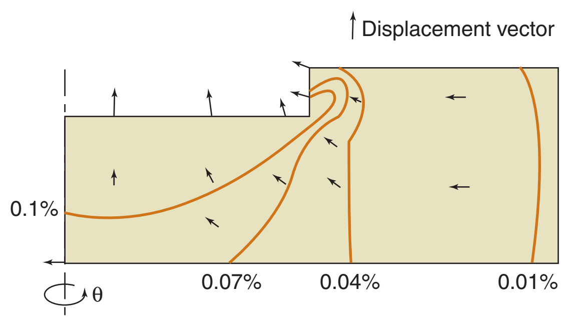
This observation leads to favoring behavioral laws for which non-linear soil behavior is well represented. As such, the model produced by Duncan and Chang [1970] had been considered for a long time one of the most popular soil behavioral laws for structural computations of retaining walls. More recently, "S"-type relations, traditionally employed for modeling dynamic soil behavior, have also been proposed for the quasi-static behavioral modeling of soils [Simpson, 1992]: as long as deformations remain small, the modulus is high; on the other hand, beyond a certain deformation threshold, the modulus dips rapidly.

To a more general extent, the soil behavior models now being championed for the structural modeling of retaining walls associate a non-linear elastic phase that accounts for average pressure, deformation and loading direction (i.e. loading/unloading) with a plastic threshold that may ultimately be strain-hardened [see, for example, Jardine *et al.*, 1991].

Moreover, special attention must be paid to calibrating the selected behavioral law in terms of stress path. The tests used for calibrating behavioral parameters are, in most instances, loading tests along the direction corresponding with the vertical to the ground, i.e.: triaxial tests and odometric tests. The

Fig. 3

Displacement isovalues in the vicinity of an excavation (computation in elasticity, for a maximum displacement of support equal to 0.2% of the wall height) [Simpson et *al., 1979]*



set of parameters identified on this basis then serve to acknowledge the soil behavior with respect to a specific loading (backfilling). These parameters do not reveal as well the soil behavior with respect to lateral loading or unloading or vertical unloading.

The use of a linear elastic law associated with a Mohr-Coulomb rupture criterion, which is typically calibrated on loading tests, thus leads to sizable ground rises of the foundation pit and pit edges during excavation simulations [Rampello *et al.*, 1992, in the case of soft clays; Mestat and Arafati, 1998, in the case of sands].

In order to resolve this problem, while continuing to rely upon simple behavioral laws, some authors have proposed identifying model zones that correspond to the various stress paths traversed. One illustration of this approach has been provided by Zhu and Liu [1994, Fig. 4], who identified four zones within the soil. In the zone lying just in front of the wall, the ratio of principal stresses decreases, whereas the opposite result is obtained both at soil depths in front of the wall and immediately in back of the wall. For these two zones, the ratio of primary stresses is increasing. For the fourth zone, i.e. deep below the soil surface in back of the wall, the ratio of primary stresses remains constant. The determination of these various zones enabled Zhu and Liu to adapt the set of soil tests serving to calibrate the computation model. Along the same lines and for behavioral laws that make use of linear elasticity, Arafati [1996] was able to distinguish, within solid zones not submitted to any eventual plastic loading, those zones submitted to an elastic loading from those submitted to an elastic unloading.

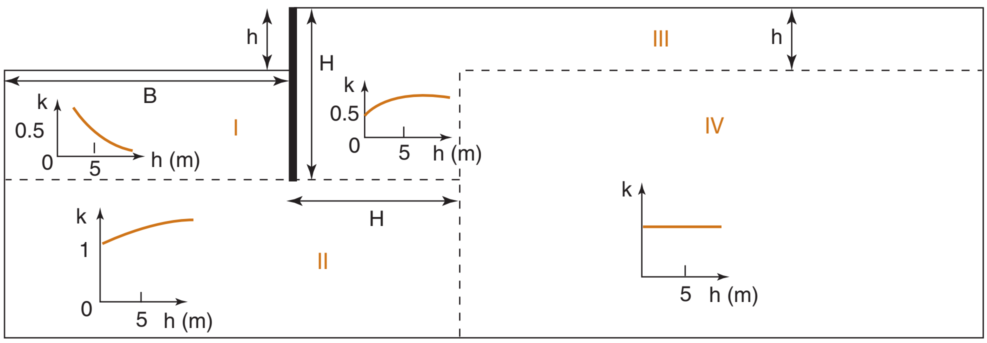


Fig. 4

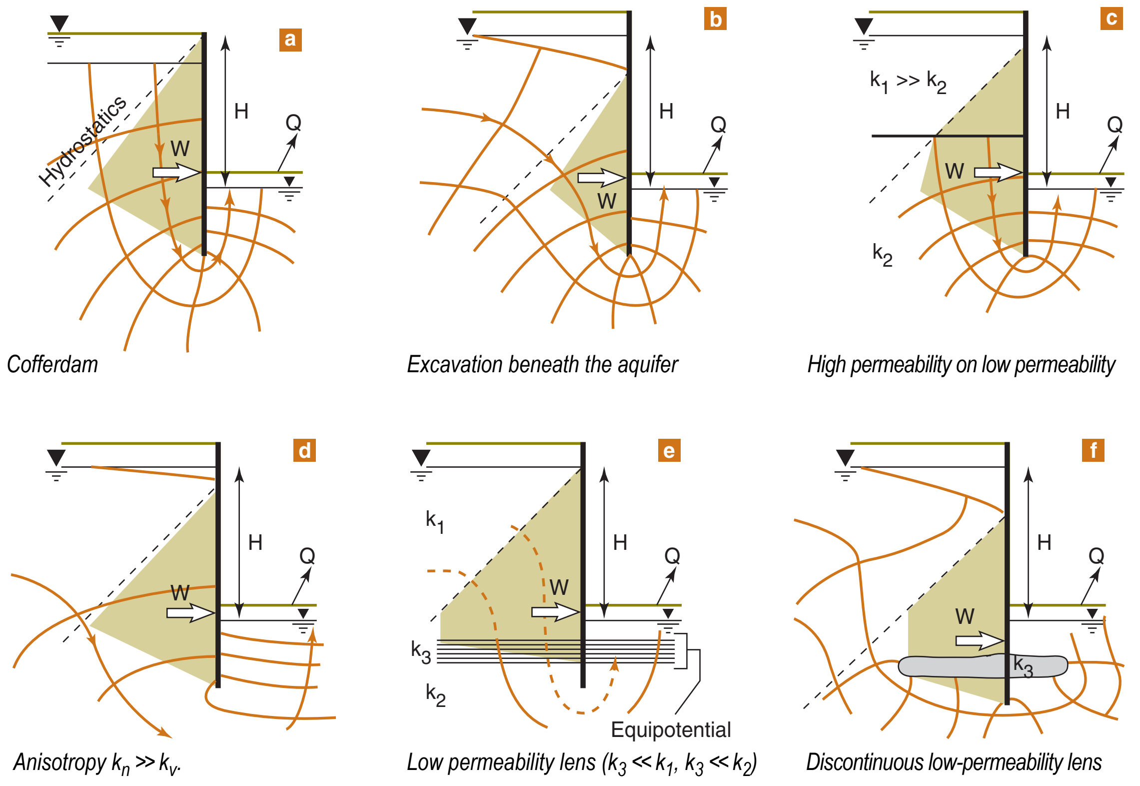
Variation in the ratio $k = \frac{\sigma_1}{\sigma_3}$ *of primary stresses in the vicinity of an excavation [Zhu and Liu, 1994]. The massif has been delimited into four zones (I through IV), with the free height of the support being designated by h and the total height by H.*

Hydrogeological conditions

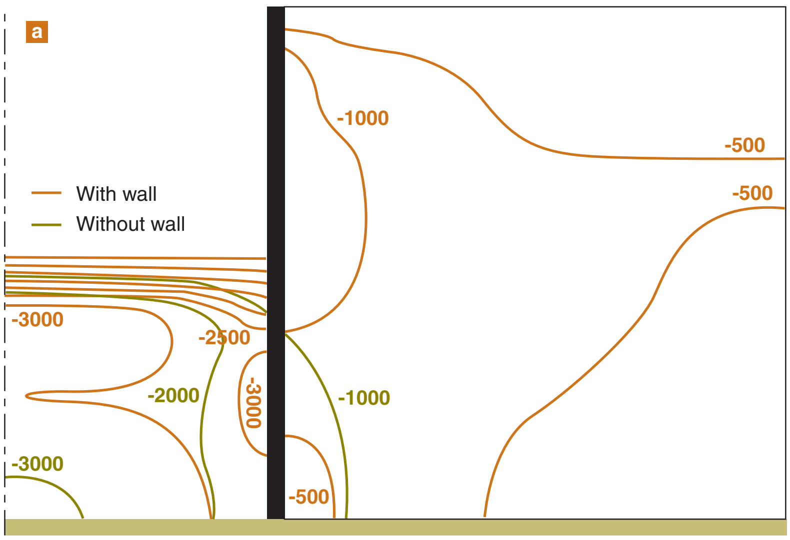
Hydrogeological conditions often prove determinant in the equilibrium of a retaining structure, as in the majority of soil mechanics problems. The finite element method has enabled accomplishing major progress at two levels.

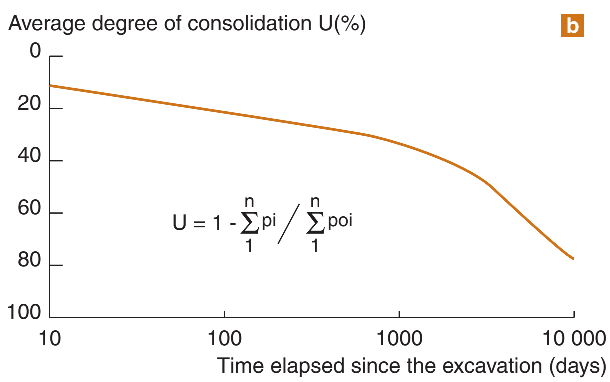
For problems in which the pressure field generated in either the steady state or transient state (depending on both the hydrogeological conditions and hydraulic characteristics of the structure) may be computed independently of mechanical aspects, the finite element method is straightforward to implement and serves to refine the analysis provided by conventional methods (see Fig. 5).

For problems where it would appear that the hydromechanical coupling must be incorporated, the consolidation theory developed from the work of Terzaghi [1925] and Biot [1941] has allowed substituting, for the conventional modeling approach to both the apparent short-term soil behavior and effective long-term behavior, an overall approach to hydromechanical equilibrium and its evolution over time. This theory constitutes an undeniable contribution as regards the comprehension and representation of the phenomena involved; nonetheless, it had remained limited to the 1D consolidation problem until the finite element computation enabled taking full advantage of this theoretical advance by offering the possibility to incorporate more complex geometries, and in particular to



■ ***Fig. 5***
Long-term equilibrium diagrams of the flow around a retaining wall for various geotechnical configurations [Kaiser and Hewitt, 1982]





■ ***Fig. 6***
Negative excess pressures at the end of the excavation (elastic massif supported by an impermeable wall) and dissipation over time; p_i denotes the existing pressure at the current instant at Point i, and p_{0i} the initial pressure; all pressure values are expressed in psi [Osaimi and Clough, 1979]

derive a solution to the consolidation problem caused by pit excavation (see Fig. 6). The benefit of this method becomes even more tangible as an excavation may indeed evolve over time from an acceptable configuration with adequate safety built in to a critical configuration once the negative excess pressures generated by the excavation have been dissipated [Holt and Griffiths, 1992; Fig. 7)]. In contrast, the opposite scenario would result from the case of an embankment on soft soil, which evolves from a configuration of critical safety over the short term to one where the level of safety is increasing over time.

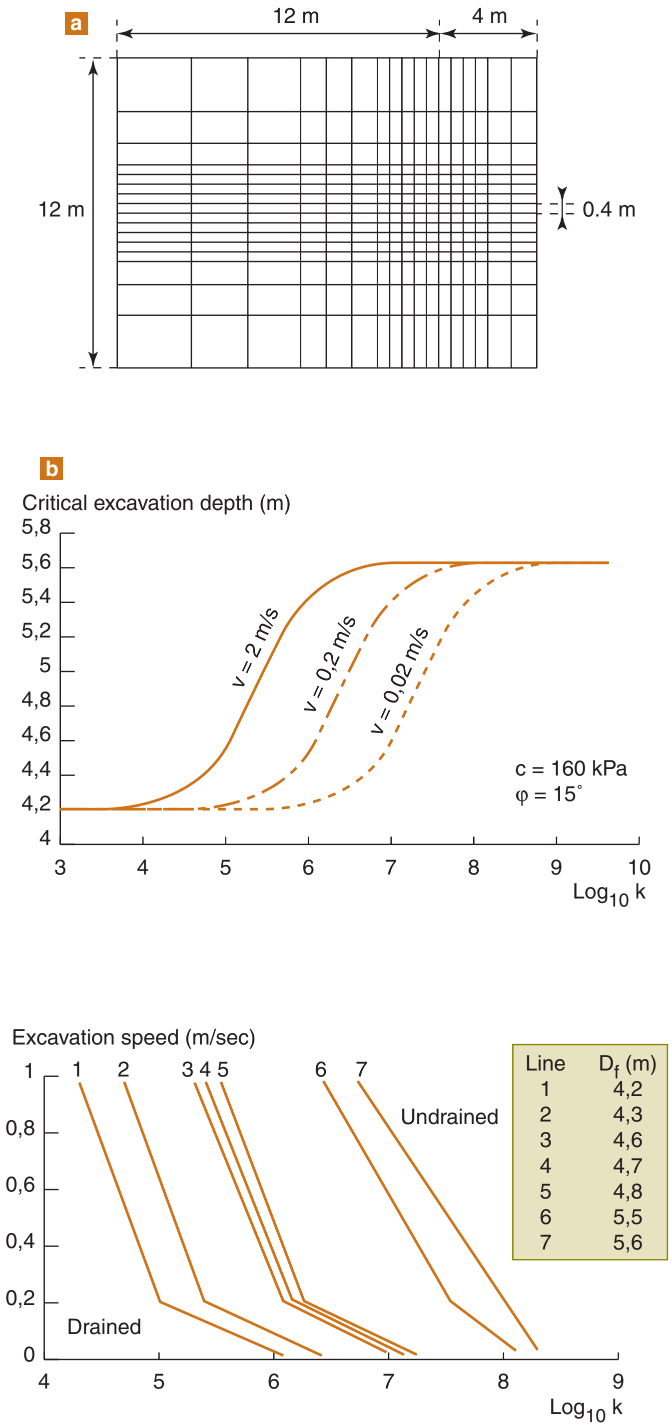


Fig. 7
Analysis of excavation conditions within an unsupported elastoplastic soil vs. permeability k and excavation speed v [Holt and Griffith, 1992]
a) Model mesh
b) and c) Critical depth D_f of the excavation obtained for various conditions of permeability and excavation speed

Wall and soil-wall interaction

Beam elements – solid elements

In order to represent the retaining wall itself, use may be made of a set of "solid" elements (i.e. elements with nonzero thickness or volume) or "structural" elements (i.e. beams in plane deformation or shells within three-dimensional computations). Application of one or the other of these modeling set-ups gets reflected, depending on the type of wall, by a more or less precise approximation on the geometry of the modeled structure: to model a sheet pile wall using solid elements, practice has dictated the need to deviate quite markedly from the actual geometry (see Fig. 8). Conversely, choosing beam elements serves to reduce the structural thickness to zero in the model, which is not necessarily very prudent for a structure of significant thickness, such as a diaphragm wall.

Moreover, from a mechanical perspective, the choice between the two modeling approaches exerts an influence, since neither of them represents the tangential stresses applied by the soil on the wall in the same fashion: the use of beam elements serves to reduce bending moments in the structure to just those moments stemming from the normal components of stresses applied by the soil. In introducing solid elements, these bending moments would be increased by moments due to the tangential components of stresses applied by the soil (see Fig. 10).

Another major distinction between the two approaches pertains to modeling the forces applied at the base of the wall. The transmission of normal and tangential forces at the wall base is simpler to understand and interpret with solid elements (even though the level of precision depends heavily on the specific mesh employed).

Composite wall panels

The 2D modeling approach for not just composite wall panels, using soldier piles and a logging, but also buttress walls serves to transform the actual wall into a plane wall with the same bending stiffness [Tsui and Clough, 1974; Fig. 9], and in most instances with the same bending stiffnesses in compression as the actual wall [Day and Potts, 1993; see Fig. 8].

Mechanical behavior

The mechanical behavior of materials composing retaining walls is typically modeled, under facility operating conditions, by means of linear elasticity. More sophisticated models may nonetheless be adopted to better ascertain the plastic behavior.

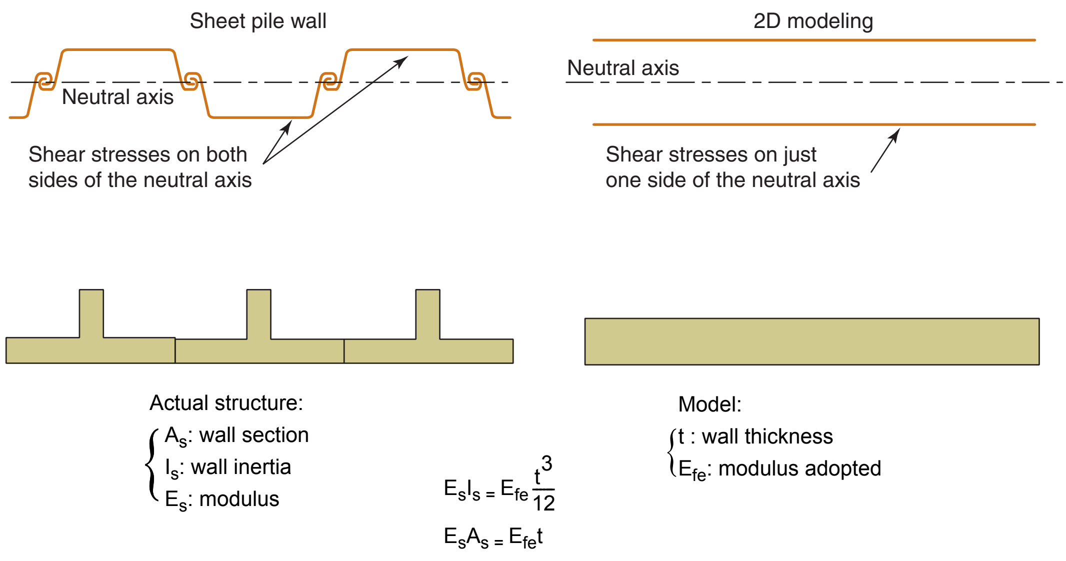


Fig. 8
Modeling of a non-plane structure by means of solid elements [Day and Potts, 1993]

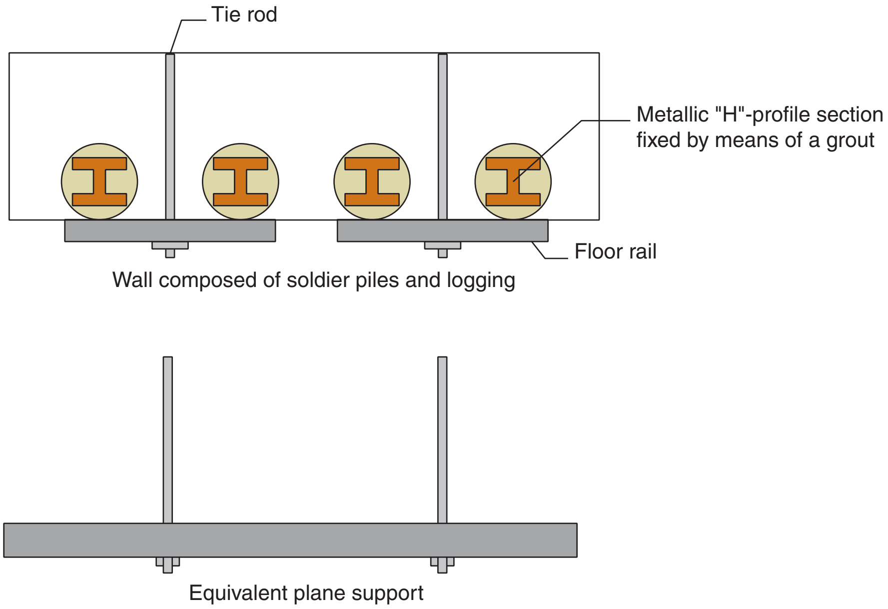
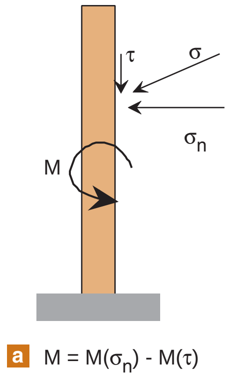


Fig. 9

Plane wall equivalent to a wall conposed of soldier piles and a logging wall: Conservation of the bending stiffness [Tsui and Clough, 1974]



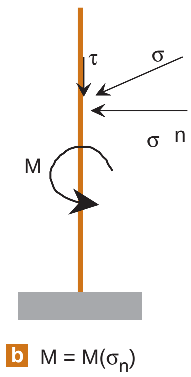
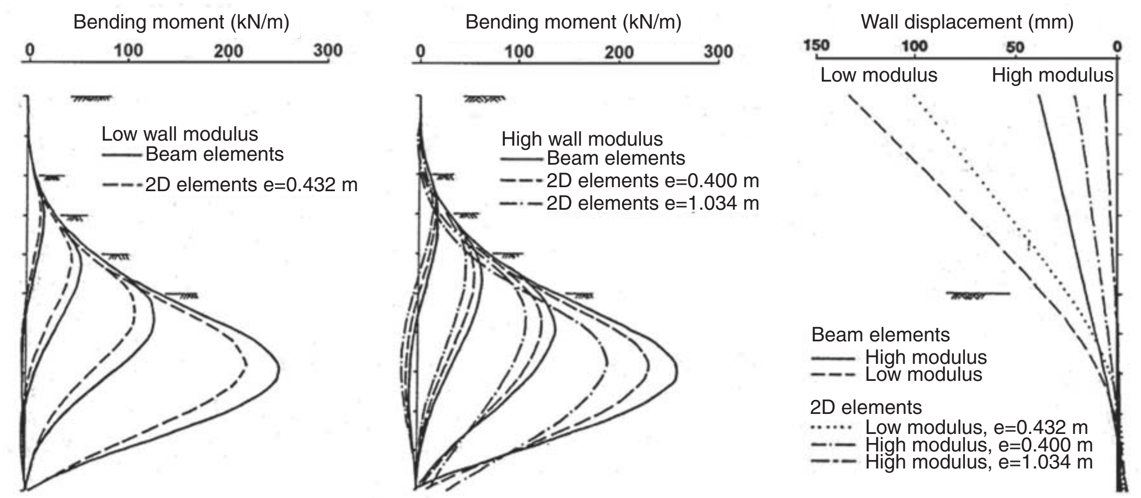


Fig. 10

Influence of the model adopted for the wall:

a) and b) Equilibrium of walls with respective nonzero and zero thicknesses

c) Results obtained by Day and Potts [1993]



c Sensitivity of the behavior of a wall simply embedded to various models

Plastic behavior exposed to sheet pile bending loads has been effectively described by Kort [2002; Fig. 11]. Smith and Ho [1992] displayed computations in which a sheet pile wall is modeled using solid elements whose behavior relies upon a plasticity criterion adjusted in order to account for the plastic moment of the steel section.

Hata *et al.* [1985; Fig. 12] incorporated, within the diaphragm wall behavior model, the appearance of cracking once a certain rate of bending had been attained. The model used stems from a beam model whose bending behavior exhibits two distinct slopes, with the second slope corresponding to the flexibility engendered in the cracked beam.

Hydraulic transmissivity

The hydromechanical modeling approach for the behavior of a soil saturated by an aquifer requires developing values of wall transmissivity. In the general case, this transmissivity is set equal to zero. Nonzero values may however be adopted [e.g. Chew *et al.*, 1997]. Transmissivity values measured *in situ* on sheet piles have been proposed by Sellmeijer *et al.* [1995].

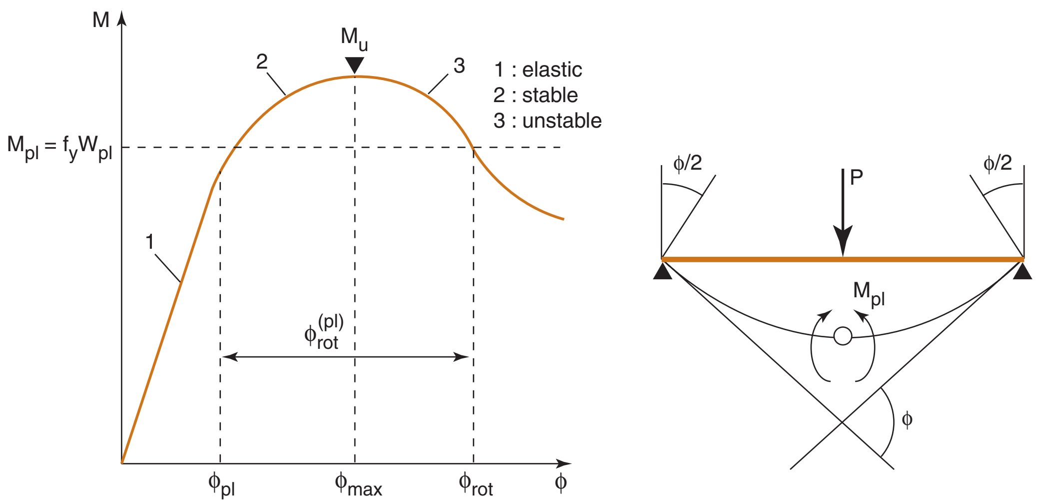


Fig. 11
Bending behavior of a sheet pile [Kort, 2002]

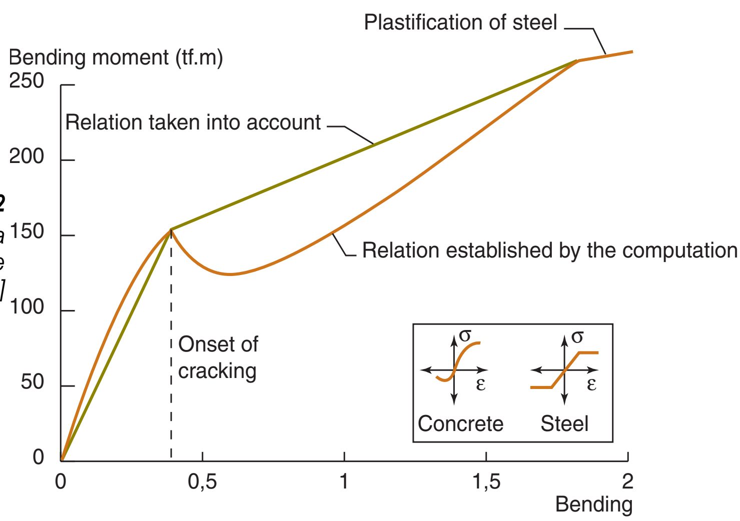


Fig. 12
Bending behavior of a reinforced concrete structure [Hata et al., 1985]

Modeling approach used for the supports

Braces

Braces tend not to be the focus of a detailed modeling approach, as the adopted model in most cases is reduced to the definition of a stiffness element (spring) at the point of applying the brace. As an initial approximation, the value of this stiffness is set equal to the compressive stiffness of the bracing elements. Nonetheless, the structural behavior observed reveals that this stiffness is most often overestimated, given both the bending deformation undergone by the bracing elements submitted to their dead weight and the behavior of the assemblies developed at the brace application points [Hata *et al.*, 1985; Duca, 2001; Fig. 13]. Furthermore, temperature variations play a major role in brace behavior. As such, a drop in temperature may lead to a significant shortening of the braces, while a temperature rise acts to increase bracing forces.

In addition, braces induce three-dimensional deformations in the structure being supported, thereby raising special problems for the modeling set-up (e.g. for inclusion in a 2D computation or for defining an appropriate mesh density on the structure around the brace support zone; see Fig. 14).

Bracing by means of floor slabs

The modeling of supports composed of floor slabs, in the case of building basements and underground parking lots, also makes use of elastic supports, just as the modeling approach for braces. The stiffness of the support is generally estimated by the compressive stiffness of the floor slab. Whittle *et al.* [1993], by virtue of proceeding with an inverse analysis of instrumentation results, has nonetheless shown that this modeling approach may be overly cursory, given the behavior of concrete: in the

Fig. 13

Temporary bracing of a covered cut, and fitting of the support zone

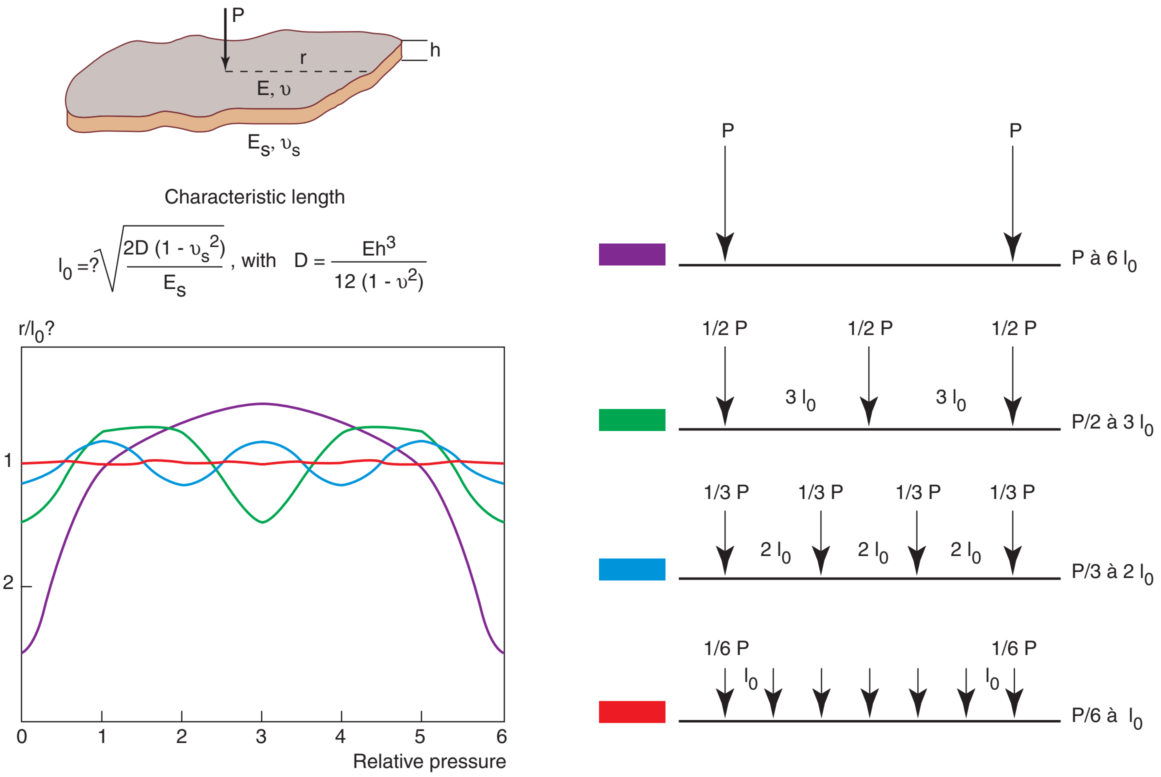


Fig. 14

Distribution of pressures on the wall vs. the distance between supports according to the theory of base plates on elastic massifs [Tsui and Clough, 1974]

specific case under study, shrinkage led to a 13-mm delamination between the floor slab and the wall, which was then exacerbated by wall displacement as excavation work was pursued. In order to incorporate this shrinkage effect, St-John *et al.* [1992] proceeded in decreasing floor slab stiffness, by taking only 20% of the theoretical stiffness.

Powrie and Li [1991] demonstrated that floor slabs were not just providing support: they also transmit a proportion of their dead weight to the retaining wall, along with the loads being applied (see Fig. 15). In the case of a covered ground cut, the type of connection adopted between the floor slab and the retaining wall (embedment or simple support on a corbel) may exert a significant influence on bending moments in the retaining wall.

Whittle *et al.* [1993] moreover indicated that temperature variations may once again serve to significantly modify the apparent stiffness of the floor slab.

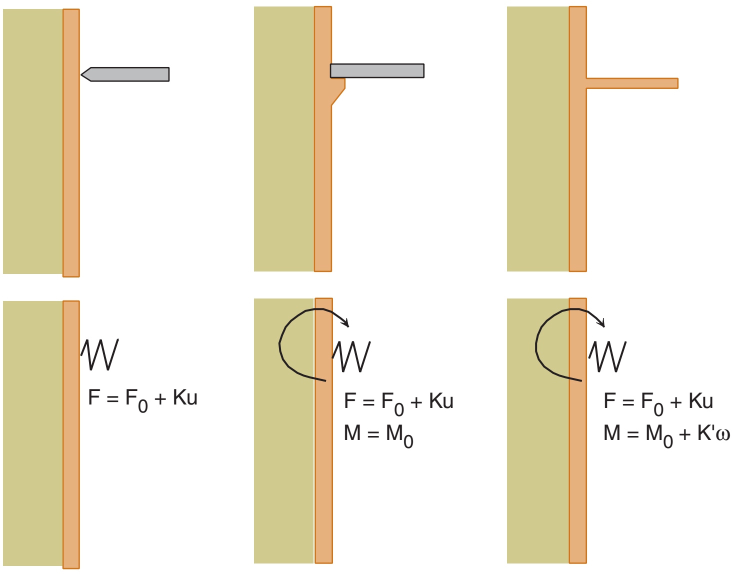
Bored and embedded anchorage tie rods

Bored and grouted tie rods are typically modeled by means of truss elements [e.g. Stroh and Breth, 1976; Day and Potts, 1991], whose stiffness is set equal to that of tie rod reinforcements and which connects the point of rod fastening to the wall with a point on the ground taken in alignment with the grouted part(see Fig. 16). The grouted part and its complex interaction with the ground have hence not been modeled.

The eventual prestressing of tie rods is obtained by virtue of the forces applied from both the wall and the point of embedment (Fig. 16b).

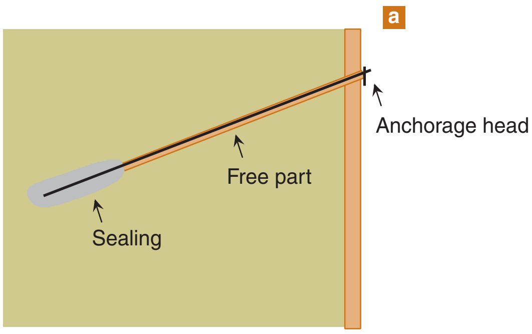
Passive tie rods working in friction

The modeling approach used for passive tie rods working in friction has required, in theory, that their three-dimensional characteristic be taken into account.

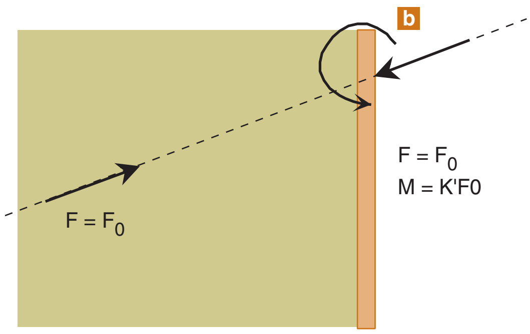


■ ***Fig. 15***

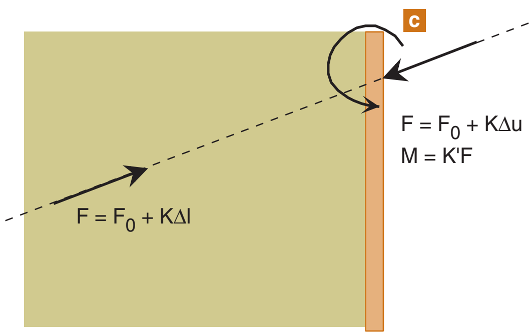
Various bracing configurations and their associated modeling set-ups



Principle of the anchorage system



Pretension modeling



Modeling for subsequent phases

■ ***Fig. 16***

Principle behind the modeling of bored, grouted and prestressed tie rods:

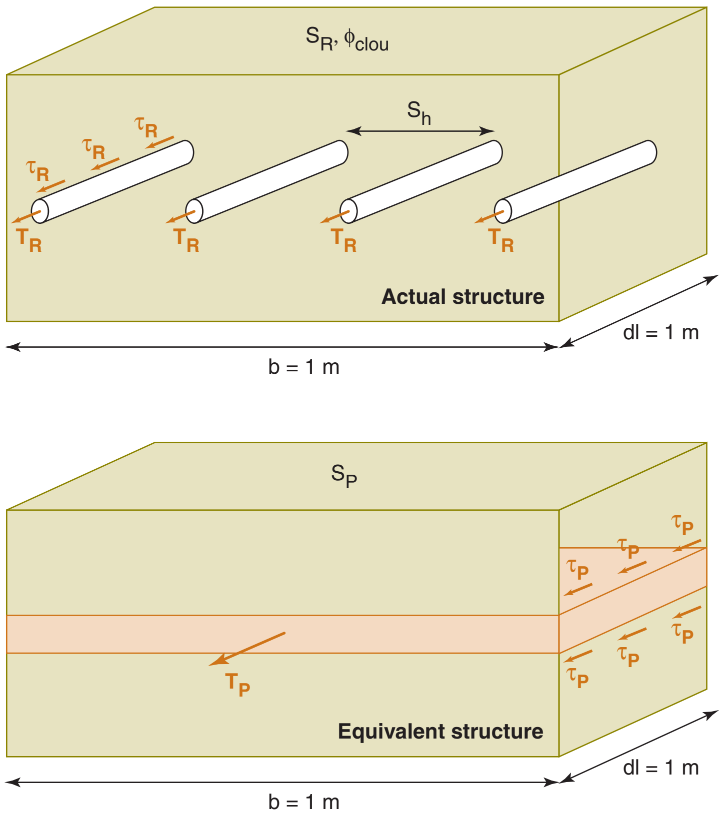
F_0: prestressing tension introduced into the tie rod

$M = K'F_0$: moment induced in the wall as a result of the incline of the tie rod and the eccentricity of the anchorage reaction with respect to the wall's neutral axis

$$K' = \frac{e}{2}\sin\alpha$$

where e denotes the wall thickness and α the angle of tie rod inclination on the horizontal

Note: The moment is only introduced into the loading model if the eccentricity of the anchorage reaction has not been modeled in the geometric model.



a) Equivalent model under two-dimensional conditions

Determination of the equivalent model:

– Conservation of the longitudinal stiffness of reinforcements: $E_{eq}A_{eq} = E_tA_t$

– Conservation of the reinforcement-soil friction (case of passive tie rods): $q_{s,eq}S_{eq} = q_{s,t}S_t$

– Conservation of the bending stiffness: $E_{eq}I_{eq} = E_tI_t$

– Conservation of soil displacements on both sides of the plate

Notations (expressed per linear meter of structure, respectively actual and modeled)

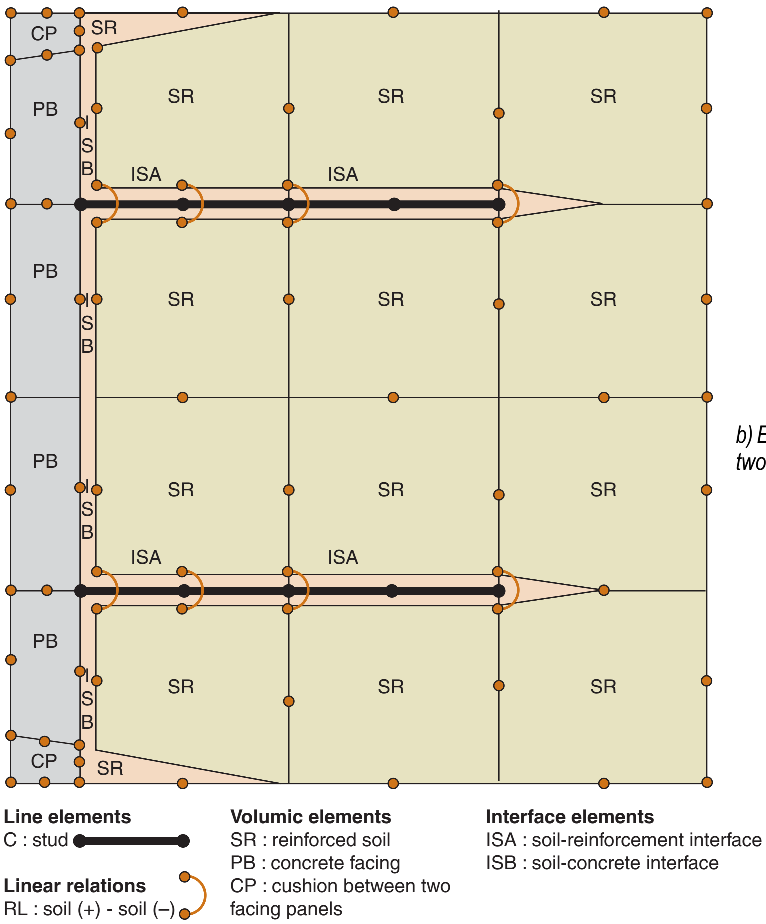
A_t, A_{eq}: tie rod sections

I_t, I_{eq}: tie rod inertias

E_t, E_{eq}: tie rod elasticity moduli

$q_{s,t}$, $q_{s,eq}$: soil-tie rod friction

S_t, S_{eq}: area submitted to soil-tie rod friction



b) Example of equivalent model mesh under two-dimensional conditions

Fig. 17

Principle behind the modeling of a wall made of reinforced earth in plane deformation [Benhamida, 1998]

Within a three-dimensional model, accounting for this type of tie rod conforms with the general principles of the modeling approach for soil-structure interactions: it is handled by modeling both the soil and the tie rods, and their interaction using special elements for representing the interface between the two [e.g. the so-called "contact" elements found in the CESAR-LCPC computation software, used notably in Al Hallak, 1999; Sellali, 1999]. The problem posed then is typically one of numerical model size, given the fact that the anchorages, of small dimension in comparison with the structure, thereby necessitate considerable refinement of the mesh.

In a plane deformation computation (which corresponds to the most common case for retaining walls), it is essential to adopt measures that enable accounting for the three-dimensional characteristic of the tie rods: this approach consists of conserving the axial stiffnesses and, if possible, bending stiffnesses, pull-out strengths, and the soil-tie rod interface stiffness, and then ensuring continuity of soil displacements on both sides of the tie rod [Benhamida, 1998; Fig. 17].

Modeling of soil-structure interfaces

The interface between soil and structures constitutes a preferential zone of deformations in which strain localization phenomena are capable of appearing: relative sliding of the soil block along the structure, delamination of both the soil and structure. Such interface behavior is commonplace in the case of retaining structures. For a simply-embedded wall supporting a sand embankment, the embankment settlement that accompanies the bending mobilization (and thus the lateral displacement) of the wall gets manifested, at the soil-wall interface, by a relative sliding of soil on the wall (see Fig. 18a). Another classical example would be that of the retaining wall of an excavation performed in highly-consistent soils: in the upper part of the excavation, soil cohesion alone is able to provide the necessary support, and the retaining wall only delaminates should it not be maintained adequately propped against the ground (see Fig. 18b).

Moreover, the soil-structure interface displays shear strength different from that of either the soil or the structure: this interface shear strength depends on an array of factors [Subba Rao *et al.*, 1996] and the value generally adopted has been reduced to a given fraction of the soil value.

The modeling of soil-ground interfaces thus requires the use of elements capable of accounting not only for these phenomena of sliding and delamination, but also for the lower shear strength.

The solution that tends to be adopted, which had initially been proposed for the modeling of discontinuities within rocky massifs, was developed by Goodman *et al.* [1968] and consists of modeling these discontinuities by means of zero-thickness elements for which two nodes may at first overlap and subsequently exhibit different displacements. In the plane case, such an element may only be submitted either to compressive forces normal to its plane or to shear forces in its plane. The model's behavior thus relies on two stiffness constants, the first relative to compressive forces and the second to shear forces.

Delamination and sliding are both simulated by means of adjusting these stiffnesses on the basis of the direction and intensity of forces applied to the element. Should the forces applied normally to the element plane be tensile forces, then the compressive and shear stiffnesses of the element are set equal to zero, thus making it possible for delamination of the initially-bonded nodes. If these forces

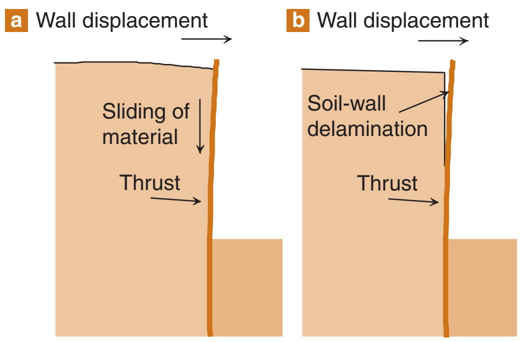


Fig. 18
Kinematics of a soil submitted to lateral decompression:
a) Case of a purely-frictional soil
b) Case of a consistent soil

are compressive, the compressive stiffness assumes a high value, thereby limiting the interpenetration of media located on both sides of the discontinuity. At the same time, the stiffness due to shear is adjusted so as to account for initial mobilization of shear and, beyond a given displacement threshold, frictional sliding of the interface.

Modeling of the building works

Modeling of the initial state of stresses

As a general rule, the initial state of stresses is unknown.

It would be legitimate to adopt simplifying hypotheses when both the geometry and loading history are simple, i.e. in the frequent case encountered in practice of horizontal ground layers with uniform loadings. The notion of a coefficient of earth pressure at rest in order to characterize the state of stresses acting upon the soil block has been introduced herein: vertical stresses are those resulting from the weight of the underlying ground, whereas effective horizontal stresses are simply correlated with effective vertical stresses by means of a coefficient that depends solely on the type of material and loading history. This notion of earth pressure at rest has moreover been the focus of many research studies and for a given type of ground, orders of magnitude are quite often known [Jaky, 1944; Mayne and Kulhawy, 1982].

Within more complex configurations, the state of stresses has not in general been determined; measurements maybe conducted, but it is commonly recognized that results deviate considerably from the actual existing stresses.

The initialization of stresses for such configurations can only be obtained therefore via simulation. Beginning with an initial situation for which the state of stresses can be determined in a straightforward manner, the primary transformations that have led to reaching the present situation are to be simulated. Such a simulation however is not always feasible, particularly when it entails complex geological phenomena. The various transformations undergone by the site are in fact not always known and modeling them may prove to be a complicated task. Furthermore, the running of such a simulation may necessitate deploying considerable computational resources.

In light of these various constraints, stress initialization for such configurations is often conducted by computing the stress field generated under the dead weight of the soil. The site history parameter thus gets significantly simplified and the stress field resulting from this type of loading may be highly dissimilar to the actual stress field.

In the presence of an aquifer, the water pressure distribution acting in the soil must also be initialized by acknowledging actual hydrological conditions. In the case where the aquifer has initially been the setting of a flow, a preliminary computation using the finite element method, depending on both the hydrogeological conditions and hydraulic characteristics of the site, will enable specifying the initial pressure field affecting the site prior to the works.

Installation of the wall

The retaining wall installation technique normally makes use of a succession of operations; it proceeds by element-specific blocks, which hence creates a three-dimensional problem. Accordingly, a diaphragm wall is built from successive panels, with the execution of each panel comprising three stages: excavation and substitution of the ground by bentonite, concreting, and hardening of the concrete. Sheet piles involve fewer stages since sheet pile panels can simply be set up one after the next. The problem however is not quite as straightforward, as these sheet pile installation operations (pile-driving, vibration-induced sinking) introduce complex and poorly-mastered phenomena. The case of combined walls no doubt engenders even greater complexity, due to its highly three-dimensional aspect.

This complexity has delayed study of the problem of retaining wall installation until just recently, while the traditionally-adopted models proceed with some major simplifications.

For diaphragm walls, the three-dimensional modeling set-up for wall construction still remains too costly to be operational [Schweiger and Freiseder, 1994; Gourvenec and Powrie, 1999; Ng and Yan, 1999; Fig. 19]. Moreover, the studies conducted have not given rise to an equivalent model that enables simulating this construction phase in plane deformation.

As regards the more widespread computations in plane deformation, a number of simplifying hypotheses have been adopted. Izumi *et al.* [1976], St-John *et al.* [1992], Whittle *et al.* [1993] all consider that wall installation does not modify the state of stresses in the soil. Schweiger and Freiseder [1994] indicate that installation of the wall is often simulated by applying the wall weight, which also happens to be the solution derived by Schweiger *et al.* [1997]. Inasmuch as the analyses performed have shown that setting up a wall tends to reduce lateral stresses in the soil, these hypotheses of no stress modification may be considered as rather conservative [Ng and Lings, 1995].

Modeling the installation of sheet pile walls or combined walls proceeds in general by means of the same simplification, with structural properties being assigned to the target elements without any modification to the existing state of stresses.

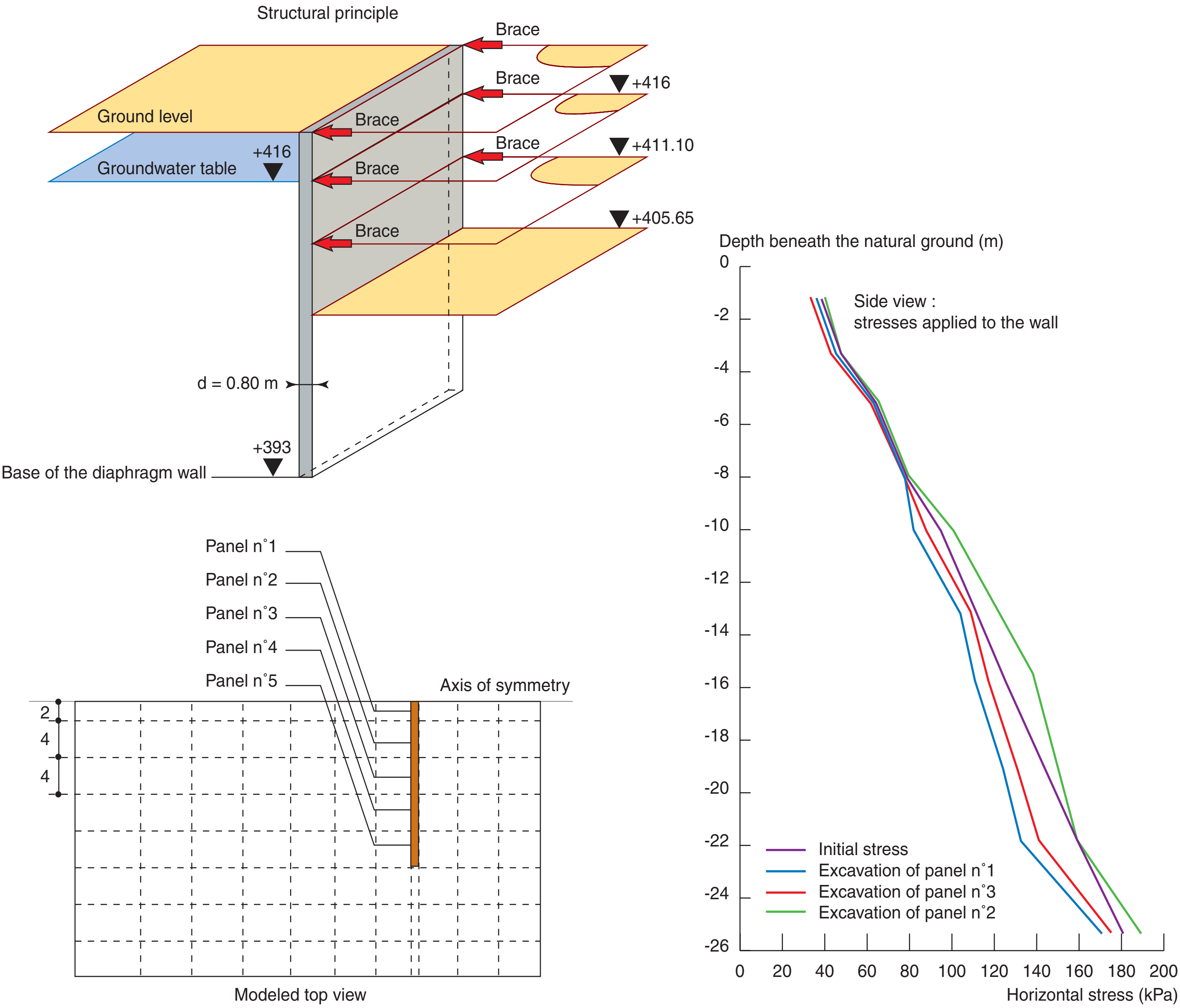


Fig. 19

Three-dimensional modeling of the installation of diaphragm walls: Sample results [Schweiger and Freiseder, 1994]

Modeling approach for backfilling stages

Backfilling consist of a common construction operation whose modeling may be approached in a variety of ways. From a general standpoint, those model zones corresponding to backfills must be defined as of the initial phase and then activated during the desired phase. This "activation" of the corresponding parts of the mesh includes ascribing a mechanical behavior and hence introducing the pertinent stiffness into the overall model, as well as incorporating the corresponding volumic loading (weight of the material).

This stress initialization within backfills is still open to debate, given the various procedures for setting up fills and compacting that are typically addressed. More complex numerical simulation processes, specifically intended to encompass stresses related to compaction in the embankment, may be developed [see, in particular, Seed and Duncan, 1986].

Modeling approach for excavation

The procedure traditionally used for modeling excavation consists of two primary features: the stiffness of the excavated elements is reduced to zero, and the stress vector is zeroed on the edge that had become a free edge. The principle employed for the purpose of canceling stresses on the now-free edge consists of applying a surface density with a force equal in intensity, yet in the opposite direction, to the stress vector operating on this edge (which, by hypothesis, has been assumed as either known over the initial stresses or to be the result of a preliminary computation): the sum of these two loadings leads to a zero loading [Chang and Duncan, 1970; Fig. 20].

This method has been criticized due to its inability to ensure that stresses are actually being reduced to zero on the free edge, since the finite element computation does not allow determining with precision the stress values at element boundaries. As such, within geometrically-complex configurations (e.g. excavation at angles), the free edge remains submitted to loadings that are often not entirely negligible.

The method proposed by Ghaboussi and Pecknold [1984] consists of directly analyzing the equilibrium of the excavated block, without utilizing the computation of nodal forces along the boundary of the excavated domain. Starting with a situation in a state of equilibrium prior to excavation between the external loading and stresses within the block, the computation focuses on searching for a new equilibrium, in light of the removal of a portion of external and internal loadings resulting from a modification in geometry. The problem to be resolved is then one of equilibrium of the "geometrically-altered" massif for which the contributions of elements corresponding to the excavation had been deleted, in terms of both stiffness and external and internal loadings.

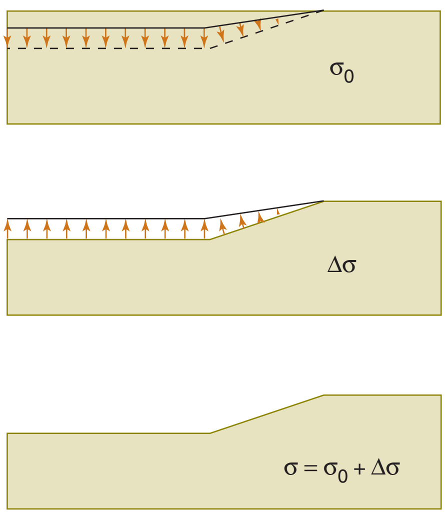


Fig. 20
Modeling of an excavation [Chang and Duncan, 1970]

This formulation may also be employed for analyzing the geometry modification provided by installing fills [Ghaboussi and Pecknold, 1984]. In this case, the stiffness matrix integrates the stiffness of newly-active elements, and the loading imbalance emanates from between the external loading that incorporates the external loading of newly-active elements and the internal loading that does not account for the internal loading of these same elements.

This numerical layout, provided by Ghaboussi and Pecknold [1984], makes it possible to proceed with the analysis of excavations within the elastic massif zones by upholding Ishihara's principle (independence of the result with respect to excavation phasing). It has since been extended to elastoplastic massifs in Borja *et al.* [1989], with massif behavior lying within the framework established by an elastoplastic law that associates linear elasticity with both Drücker-Prager and Von-Mises criteria. Borja [1990] repeated the exercise, with massif behavior being governed by the modified Cam-Clay model. This layout has also been coupled with the problem of free-surface flow into the soil [Hsi and Small, 1992a, 1992b; Borja, 1992], in a way that allows for the coupled analysis of the impact of ground excavation and modifications contributed to aquifer conditions by virtue of the excavation.

Modeling in plane deformation, axisymmetrical as well as three-dimensional

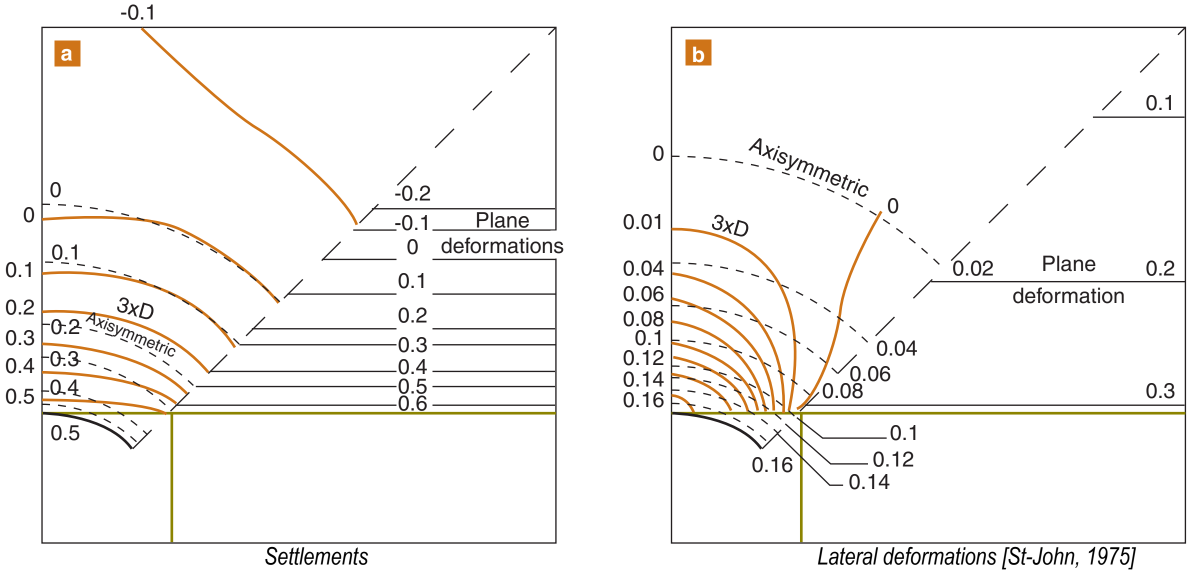
The limitations of conventional computation methods have, until the present time, led to simplifying retaining structure problems into plane problems. The finite element method does not, at least in theory, display the same set of limitations given that it serves to analyze three-dimensional configurations. From a practical standpoint however, it would be useful to observe that computing capacities, in terms of both computational power and storage volume, have most often confined this method's application to two-dimensional configurations.

Several aspects nonetheless are capable of limiting the relevance of modeling retaining structures and excavations in plane deformation:

- for very long structures, such a hypothesis is more typically acceptable, at least for structural sections far enough from eventual singularities; for shorter structures however, this hypothesis is in most instances not admitted (see Fig. 22);
- the various stages in the construction process do not pertain simultaneously to all structural sections, but instead proceed in a "progressive" manner, generally in the structure's longitudinal direction. Such is the case in particular with retaining wall installation (diaphragm walls or sheet piling), as well as with earthwork and placement of the supports. Accordingly, all tie rods of a given set of anchorages are not pre-tensioned simultaneously, but rather one after another, in accordance with a phasing program linked to worksite organization. The application of loads therefore is not strictly two-dimensional;
- certain structural components are not plane, but tend more towards distributed elements in a discrete manner: structural supports overall, and tie rods and braces, plus foundation elements adjacent to the structure.

Under these conditions, the choice of computation model takes on special importance. It would thus appear that an axisymmetrical model could be preferable to a model in plane deformation for the purpose of evaluating the behavior of a pit featuring a small slenderness ratio in the plane dimension. Figure 21 [St-John, 1975] superimposes the isovalues of movements caused in the vicinity of a square pit, as computed under three-dimensional conditions, of a cylindrical pit, as computed under axisymmetrical conditions, and lastly of a linear pit, as computed in plane deformation (with all three pits exhibiting the same transversal dimension). This figure shows that the axisymmetrical model yields a close approximation of the soil movements caused by the square pit, outside the immediate vicinity of the pit, whereas the linear pit lies at the origin of more sizable movements than either the square or cylindrical pit with the same transversal dimensions.

From a more general perspective, the pertinence of modeling in-plane deformation for analyzing rectangular pits, depending on their level of slenderness, has been studied [Ou *et al.*, 1996; Moormann and Katzenbach, 2002; Fig. 22]. These efforts have illustrated the limitations of the plane deformation analysis of structural behavior and attest to the potential offered in the field of geotechnical engineering by three-dimensional applications of finite element methods, as inspired by their current widespread application in the field of industrial mechanics.

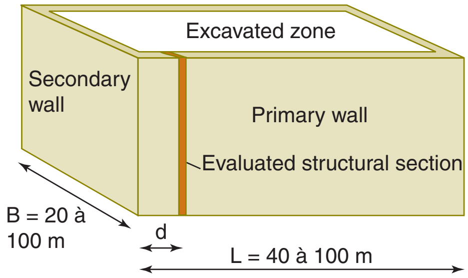


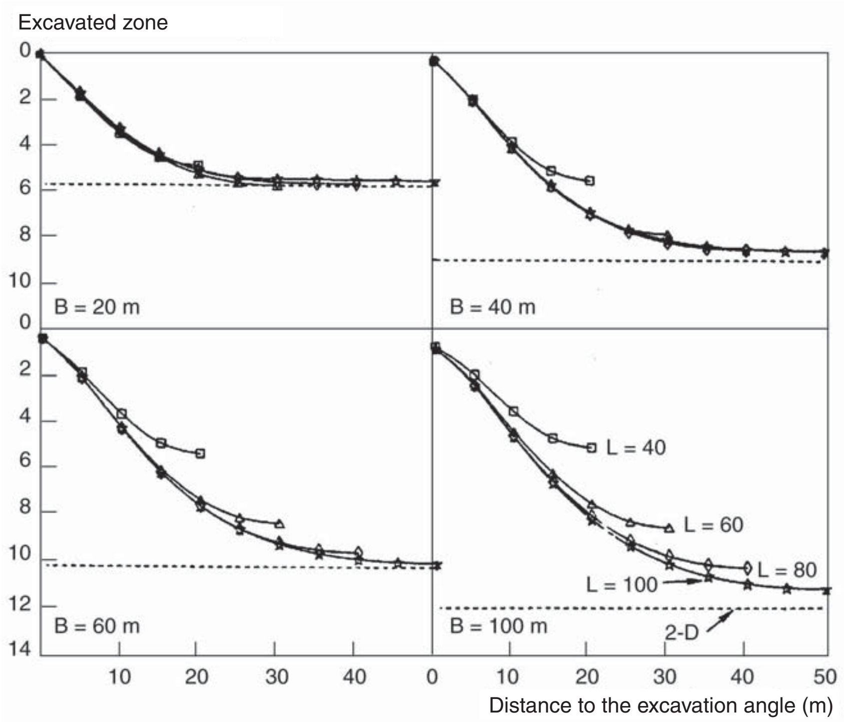
■ ***Fig. 21***

Comparison of computation results of movements caused at the soil surface by three pits of the same transversal dimension (computations conducted in plane deformation for a linear pit, in axisymmetry for a cylindrical pit, and in 3D for a square pit). The pit is not supported and has been excavated within an elastic massif.

■ ***Fig. 22***

Example of three-dimensional characteristic: lateral displacement of primary wall vs. its length L for various widths B of the pit [Ou et al., 1996]





CONCLUSION

The bibliographical review presented serves to emphasize the wide extent of responses being contributed today, using the finite element method, to the diversity of questions raised by the modeling of retaining structures. This undertaking has, conversely, also reflected the major weaknesses inherent in conventional methods for computing such structures, which in the end can only claim to account for a small number of aspects within such problems.

As a conclusion, it seems possible for us to distinguish two aspects of this modeling approach to retaining structures, for which the finite element method casts an entirely new light compared with more conventional methods: 1) the often three-dimensional structural behavior, and 2) consolidation phenomena related to excavations.

Three-dimensional behavior, with respect to both structural geometry aspects and the construction techniques employed, may only be represented by a plane model under a set of very specific conditions. This most certainly constitutes a major limitation of the methods used in more traditional contexts and the three-dimensional computation of structures by means of the finite element method, now made possible thanks to progress in computing power, is now poised to develop much more extensively.

Consolidation phenomena have been studied exhaustively in cases where they have been incited by embankment construction. The investigation of these phenomena, in contrast, has remained in the nascent stages in cases where they have been incited by excavations, even though this latter situation is encountered just as frequently, as regards both long-term structural stability and deferred deformations capable of arising following the works. No simple-to-implement analytical method for consolidation phenomena thus exists for application to excavation supporting walls. The finite element method remains the only feasible analytical method and should be recognized as such over time.

As for the difficulties still encountered during implementation of the finite element method for the modeling of retaining structures, one aspect has apparently yet to be adequately resolved. The finite element solutions that enable representing within the same model a plane or volumic structure (ground, wall) along with a line structure (tie rods) have not reached a very satisfactory level and the modeling of anchored retaining structures must therefore progress still further.

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