

# Influence of test loading rate and anisotropic specimen consolidation during cyclic triaxial testing

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## ■ ABSTRACT

Two practical aspects of undrained cyclic triaxial test procedures are examined. The first one concerns the rate of loading, which is always significantly greater than that applied during monotonic loading tests on silty and clayey soils. The second procedure concerns the level of static stresses applied during cyclic loading, in order to explain that the yield strength criteria are more readily reached in tension during a strictly alternating test protocol. A summary of testing procedures is presented, along with examples of test results, before offering a few concluding remarks.

## Influence de la vitesse d'essai et de la consolidation anisotrope de l'éprouvette dans les essais triaxiaux cycliques

### ■ RÉSUMÉ

*Deux aspects pratiques de la réalisation des essais triaxiaux cycliques non drainés sont examinés. Le premier concerne les vitesses de chargement, qui s'avèrent largement supérieures aux vitesses appliquées pendant les essais à chargement monotones sur les sols limoneux ou argileux. Le second aspect concerne le niveau des contraintes statiques pendant les cycles, pour expliquer que les critères en résistance cyclique sont atteints préférentiellement en extension pendant les essais strictement alternés. Les procédures d'essai sont évoquées sommairement. Des exemples de courbes d'essais sont présentés et commentés ensuite.*

## INTRODUCTION

Two practical aspects during the execution of undrained cyclic triaxial tests will be discussed herein on the basis of experimental data. To proceed with this discussion, three series of undrained cyclic tests were conducted in the laboratory. The first relates to a clayey silt tested in its natural state, with the aim of observing the loading rate effect. The other two pertain to a sand specimen reconstituted in the laboratory, on which the range of static stress levels was applied during the cycles.

The issue of cyclic loading rate is an important one since it influences both the test duration and quality of results, in particular as regards pore pressure measurements. This issue is not raised with clean sands, for which cyclic frequencies ranging from 0.1 to 0.5 Hz are most often applied. Such frequencies however result in loading rates that exceed, sometimes by a wide margin, the rates prescribed in triaxial procedures for monotonic loading tests on silty or clayey soils. According to these procedures, loading rates are limited in order to maintain pore pressure fields relatively homogeneous in undrained triaxial specimens. This issue of cyclic loading rates thus also focuses on the quality of triaxial testing.

The second problem encountered concerns the effect of initial stress state on the cyclic soil strength. This topic has been addressed in many experimental studies using either the triaxial device or other

systems (simple shear, hollow cylinder). In practice, cyclic testing procedures call for performing undrained cyclic loadings on pre-consolidated specimens under isotropic stresses. In this case, the cyclic triaxial loadings are strictly alternated. Experience has shown that breaking most often occurs in tension. At sites of recent, normally consolidated deposits, the initial stress states are not isotropic but instead located on the compression side. Encroaching upon the tension side during alternating cycles in the laboratory is therefore penalizing for soils, resulting in the measured cyclic strength having to be estimated by default.

## TEST PROCEDURE

### ■ Loading mode

In accordance with their design, the cyclic tests presented herein have been conducted with very slow loading rates. The testing procedure entails implementation of standard triaxial equipment. The undrained loadings (CU) are carried out in an “FR” mode, with control of both the axial force ( $F$ ), which varies between two fixed bounds, and a displacement rate ( $R$ ) that is constant and imposed by the triaxial press.

Specimens are saturated with a back pressure applied in incremental steps before being consolidated using an effective isotropic pressure  $p_c = 100$  kPa. Next, the force-controlled cycle sequences are performed at different loading rates  $v$ , for various amplitudes  $\Delta q_y$  of the deviator  $q$  and a range of average values  $q_{ave}$ . As such, the cycles are defined by an axial force  $F$  that oscillates between two limits, corresponding approximately to the two deviators  $q_{min}$  and  $q_{max}$  (note: this approximation depends on the specimen section correction applied during the cycle sequence). The cyclic amplitude is expressed as  $\Delta q_y = (q_{max} - q_{min})/2$ , while the average deviator is  $q_{ave} = (q_{max} + q_{min})/2$ .

In this type of loading with constant rate  $v$ , the deviator alternates over time (i.e.  $q$  alternately positive and negative, with amplitude  $\Delta q_y$  remaining less than the average deviator  $q_{ave}$ ,  $\Delta q_y < |q_{ave}|$ ) or else ripples (i.e.  $q$  remains positive, with amplitude exceeding the average deviator,  $|q_{ave}| < \Delta q_y$ ). The duration of undrained cycle sequences depends on the selected rate  $v$ , ranging from a few hours to some 100 hours to perform 10 to 20 cycles.

Cycle sequences are expressed by the following variables:  $\epsilon_a = \Delta h/h_s$ , axial strain ( $h_s$ : specimen height after consolidation);  $p = (\sigma'_a + 2\sigma'_r)/3$ , average effective pressure;  $q = \sigma'_a - \sigma'_r$ , the deviator; and  $u$ , pore pressure measured at the base of the specimen with  $\sigma'_a$  and  $\sigma'_r$  the effective axial and radial stresses, respectively.

Upon completion of the cycle sequences, the undrained cyclic strength of the soil is defined by a criterion that consists of determining the cycle number at which a given level of axial strain appears. In the present case, this level is evaluated by the simple axial strain amplitude  $\epsilon_{aySA}$  as equal to 2.5% in absolute value terms, whether in compression or tension.

Triaxial tests with monotonic loadings were also conducted to accompany the cyclic tests.

### ■ Choice of monotonic test rates

In common practice, standards have set forth the rules for choosing loading rates to be imposed upon triaxial devices (e.g. AFNOR, 1994). These rules stem from a theoretical approach developed during the 1950's (Bishop and Henkel, 1964).

During the series of undrained triaxial tests, non-homogeneity of the pore pressure field inside the specimen is ascribed to the heterogeneity of stresses and strains, caused among other things by the hooping effect on triaxial heads (Bishop and Henkel, 1964). This heterogeneity may be accentuated by the loading rate effect. A return to homogeneity depends on: soil permeability, its degree of induration or overconsolidation, specimen dimensions, and loading rate.

A ratio of pore pressure equalization was defined by the expression:  $1 - \Delta u / \Delta u_0$ , where  $\Delta u_0$  is the difference in pore pressure between the center of the specimen and its ends for a given axial strain  $\varepsilon_{af}$  and a slow loading; and  $\Delta u$  is this same difference at time  $t_f$  for the same level of strain during actual loading at a constant rate. Time  $t_f$  must be chosen so as to ensure that the equalization rate reaches 100%.

A theoretical relationship was established by Gibson and Henkel (1954) to correlate the equalization rate with both a degree of consolidation  $U_f$  and a time factor  $T_f$ , the latter of which is indicated in **Table 1**.

**Table 1**  
Values of time factor  $T_f$  that allow obtaining a given equalization rate as a function of drainage conditions

Equalization rate	Lateral drainage	Only axial drainage
95%	0.071	1.67
99%	0.36	8.50

The increase from 95% to 99% in equalization rate raises the time factor by a ratio of approx. 5. In practice, the calculation of durations  $t_f$  for triaxial CU tests with monotonic loading involves both the level of strain  $\varepsilon_{af}$  and time factor  $T_f$ . Time  $t_f$  is given by the following relation, in writing:  $h^2 / c_v = t_{100} / \rho$ :

$$t_f = T_f h^2 / c_v = T_f t_{100} / \rho$$

where:  $\rho$  is the coefficient that depends on drainage conditions at the specimen boundaries ( $\rho = \pi, \pi/4, \pi/64$ , etc.),  $t_{100}$  the time of end of primary consolidation,  $h$  the half-height of the triaxial specimen, and  $c_v$  the soil consolidation coefficient. Hence, the maximum displacement rate  $v_{max}$  to offer as a guideline for the triaxial machine would be:

$$v_{max} = \Delta h_f / t_f = h_0 \varepsilon_{af} / [T_f t_{100} / \rho] = h_0 \varepsilon_{af} / (a t_{100})$$

where  $h_0$  is the specimen height and  $a = T_f / \rho$ . This coefficient  $a = T_f / \rho$  rises from 2.1 to 10.8 as the equalization rate climbs from 95% to 99% in a specimen drained at both its ends. In most cases, the advised rate is 95%. Choosing a rate of 99% extends the test duration fivefold.

### ■ Cyclic testing rates

During a cyclic triaxial test with a sinusoidal loading, frequency  $f$ , period  $T$  and amplitude  $\Delta q_y$ , the maximum loading rate equals:

$$(d\varepsilon_a/dt)_{max} = 2\pi \Delta q_y / (T E_u)$$

where  $E_u$  is the undrained Young's modulus of the soil.

In the example of a soft clayey soil with a modulus value of  $E_u = 50$  MPa and a cyclic test of small amplitude  $\Delta q_y = 10$  kPa, the equivalent monotonic loading rates are indicated in **Table 2** as a function of frequency  $f$  and period  $T$ , for specimens 100 mm high. It appears that cycle rates within the frequency range of 0.1 to 1 Hz are substantially greater than those of the monotonic CU tests, which are often on the order of 0.010 mm/min (10  $\mu$ m/min) for fine-grained silty or clayey soils.

**Table 2**  
Examples of maximum loading rates during sinusoidal cycles

f (Hz)	10	1	0.1	0.01	0.001
T (s)	0.1	1	10	100	1000
v (mm/min)	75.4	7.54	0.754	0.075	0.007

## LOADING RATE EFFECT

### ■ Tests

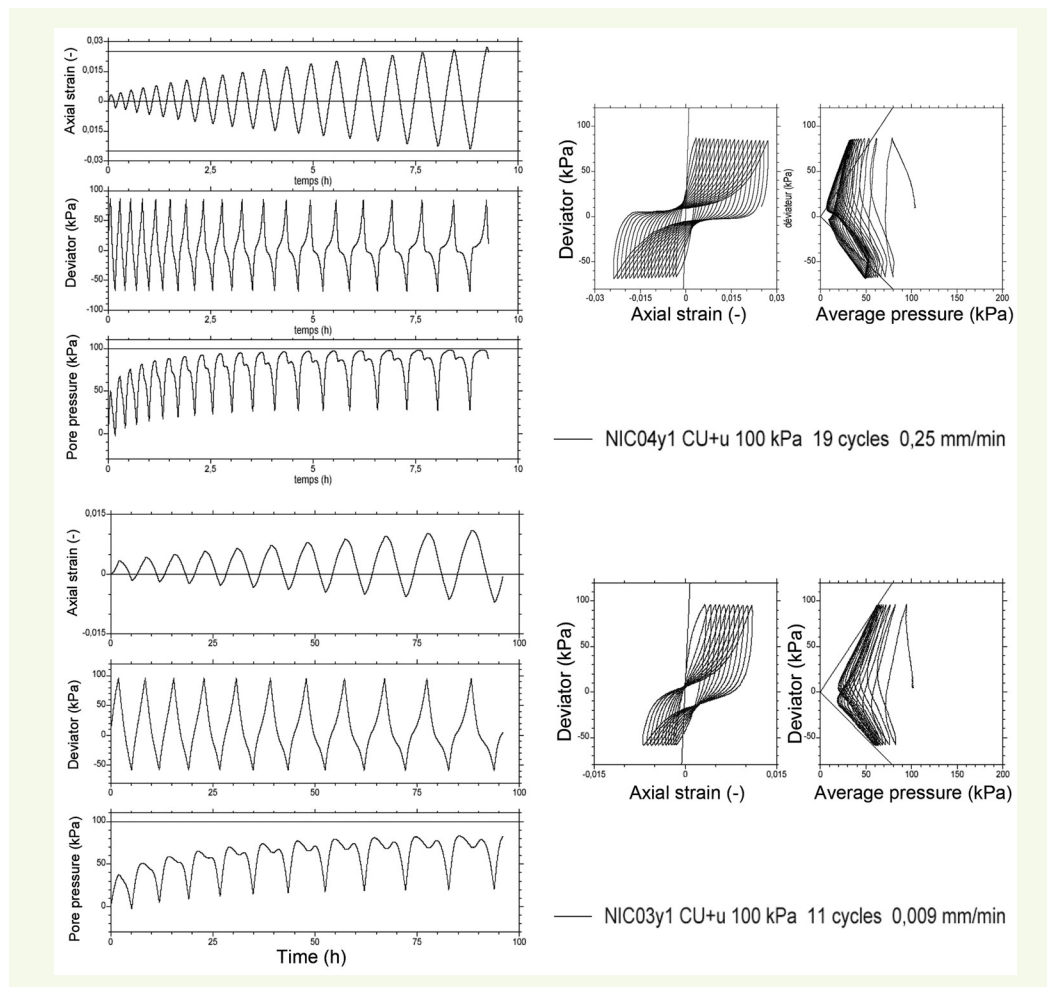
The tested soil was extracted from core samples bored at a depth of 24 m. The material was a blackish-brown clayey silt. The triaxial specimens were manually cut with a bench grinder on a section extracted from a core sample (specimen diameter: 50 mm, slenderness ratio: 2).

The specimens were consolidated by being exposed to an effective isotropic pressure  $p_c$  equal to 100 kPa. Next, the cycle sequences were carried out in “FR” mode (defined above) for various rates  $v$  included between 0.009 and 0.25 mm/min, in the presence of identical deviator amplitudes  $\Delta q_y = 77$  kPa and strictly alternating cycles ( $q_{ave} \approx 0$ ). The duration of these undrained cycle sequences ranged from 9 to 96 hours.

Test results were interpreted in terms of effective stresses.

### ■ Results

Soil responses during the two cyclic CU tests are shown in **Figure 1**, along with the curves  $(t, \epsilon_a)$ ,  $(t, q)$ ,  $(t, u)$  and  $(\epsilon_a, q)$  and effective stress paths  $(p, q)$ . The first test was performed at the rate



**Figure 1**  
Cyclic triaxial CU tests conducted at two distinct loading rates in “FR” mode - left: variation curves of the axial strain  $\epsilon_a$ , deviator  $q$  and pore pressure  $u$  vs. time; right: stress-strain curves  $(\epsilon_a, q)$  and effective stress paths  $(p, q)$

$v = 0.25$  mm/min (19 cycles in 9 hours), while the second used the rate  $v = 0.009$  mm/min (11 cycles in 96 hours). The rate applied during this test is comparable to that prescribed for slow undrained monotonic tests.

The curves  $(t, \epsilon_a)$  display a very gradual shape, without any abrupt trend, which reflects a typical response for a clayey soil. The increase in pore pressure is very slow during the second test. In the effective stress plane  $(p, q)$ , the stress paths do not reach the origin. During the first test, these paths  $(p, q)$  “surpassed” the Coulomb compression and tension lines at both the top and bottom of the cycle.

The Coulomb compression and tension lines depicted in planes  $(p, q)$  have been determined via triaxial CU tests with slow monotonic loadings. Their slopes equal respectively:  $M_c = 1.51$  and  $M_c = -1.01$ , with the corresponding angle of friction  $\phi' = 37.2$  degrees.

In comparison with slower loadings, these high cycle rates tend to shift the effective stress paths towards lower average pressures. The slopes  $M_c$  and  $M_c$  have thus been overvalued in absolute terms. In the special case of the two cyclic tests presented herein, the angle of internal friction has been overvalued by nearly 10 degrees.

## EFFECT OF THE STATIC STRESS LEVEL

### ■ Tests

The sand sample was extracted by boring at a depth of around 3 m. This material is a slightly silty sand with particle contents of:  $C_{200\ \mu\text{m}} = 62\%$ ,  $C_{80\ \mu\text{m}} = 26\%$ ,  $C_{20\ \mu\text{m}} = 17\%$ ,  $C_{2\ \mu\text{m}} = 5\%$ , and a  $D_{\text{max}} = 0.5$  mm. Both monotonic and cyclic triaxial CU tests were undertaken on intact soil, yet the number of tests remained limited due to the reduced availability of core samples.

Specimens from the two series presented in this work were reconstituted in the laboratory (diameter: 50 mm, slenderness ratio: 2). As part of this reconstitution step, the initially dry sand was wetted very slightly (water content  $w = 2\%$ ) and then poured into a mould in three layers and tamped in order to yield the desired density (wet tamping). The average void ratio of these reconstituted specimens equals  $e = 1.02$  approx. The minimum and maximum density measurements reveal that the density index of this reconstituted sand equals  $I_D = 48\%$ .

Cycle sequences were performed in “FR” mode at a constant rate  $v = 0.05$  mm/min, after isotropic consolidation of the specimens at the effective pressure  $p_c = 100$  kPa.

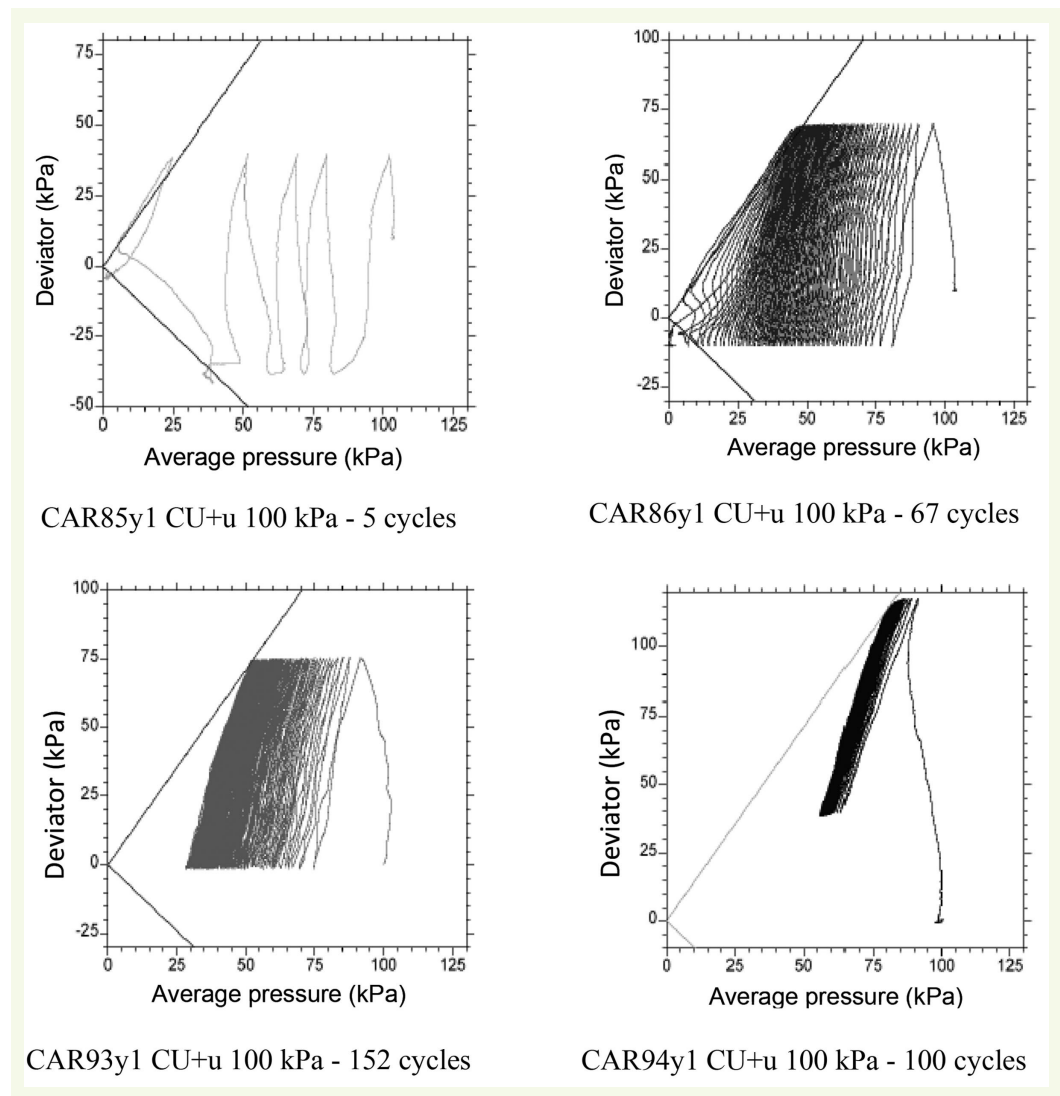
These cycle sequences can be characterized by identical amplitudes  $\Delta q_y$ , such that  $\Delta q_y / 2 p_c \approx 0.29$  for series 1 and  $\Delta q_y / 2 p_c \approx 0.20$  for series 2. The average static values  $q_{\text{ave}}$  differ. The undrained cycle sequence duration extended from 7 to 66 hours.

### ■ Results

**Table 3** lists the cyclic loading characteristics during four cyclic CU tests from series 2.  $N$  is the number of cycles and  $N_{2.5\%}$  the number of cycles required to satisfy the criterion  $\epsilon_{\text{ay SA}} = 2.5\%$ . **Figure 2** shows the effective stress paths  $(p, q)$  followed during these tests, which were respectively: strictly

Specimen	$q_{\text{ave}}$ (kPa)	$\Delta q_y$ (kPa)	$N$	$N_{2.5\%}$	$q_{\text{ave}}/2p_c$
CAR85	1	39	5	4	0.005
CAR86	29	39	67	6	0.145
CAR93	37	38	152	< 152	0.185
CAR94	78	40	77	77	0.390

**Table 3**  
Cycle characteristics  
for four series  
2 specimens ( $p_c = 100$  kPa;  
 $\Delta q_y / 2 p_c \approx 0.20$ )



**Figure 2**  
 Cyclic triaxial CU tests conducted at two distinct loading rates in “FR” mode - left: evolution curves of the axial strain  $\varepsilon_a$ , deviator  $q$  and pore pressure  $u$  vs. time; right: stress-strain curves ( $\varepsilon_a, q$ ) and effective stress paths ( $p, q$ )

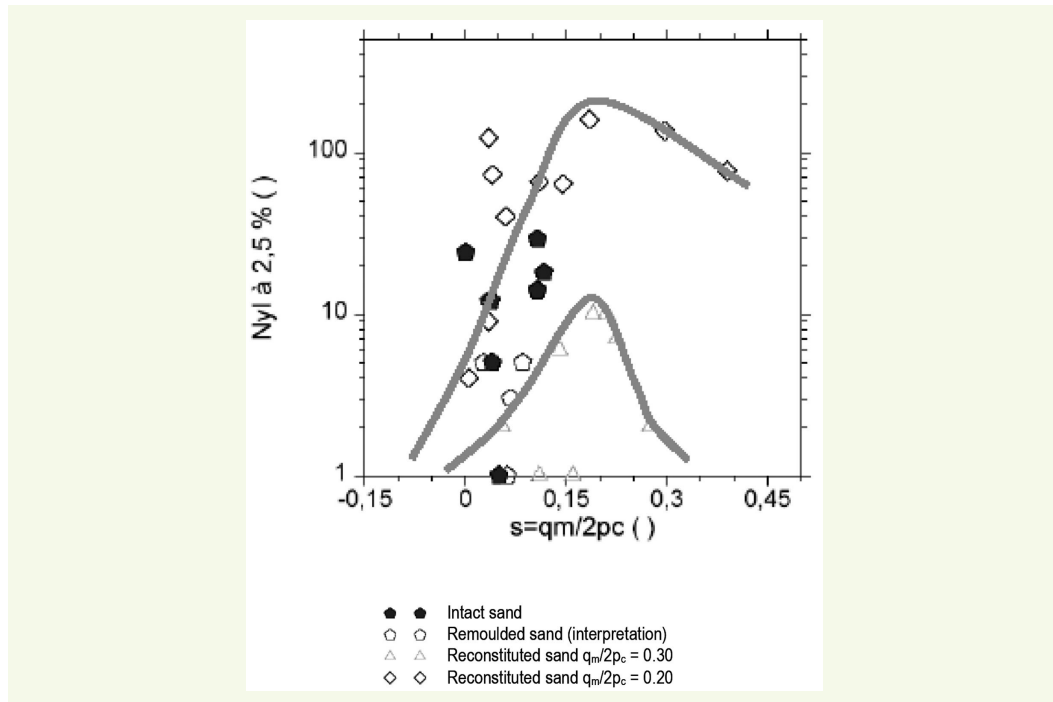
alternated, then alternated in compression or rippled, for the same amplitude  $\Delta q_y$ . This criterion is satisfied in 4 cycles during the CAR85 test, which is of the strictly alternating type.

During ripple tests, the effective stress paths ( $p, q$ ) stabilize quickly. Nonetheless, the  $\varepsilon_{ay SA} = 2.5\%$  criterion is satisfied earlier during the CAR94 test performed under the highest static stress.

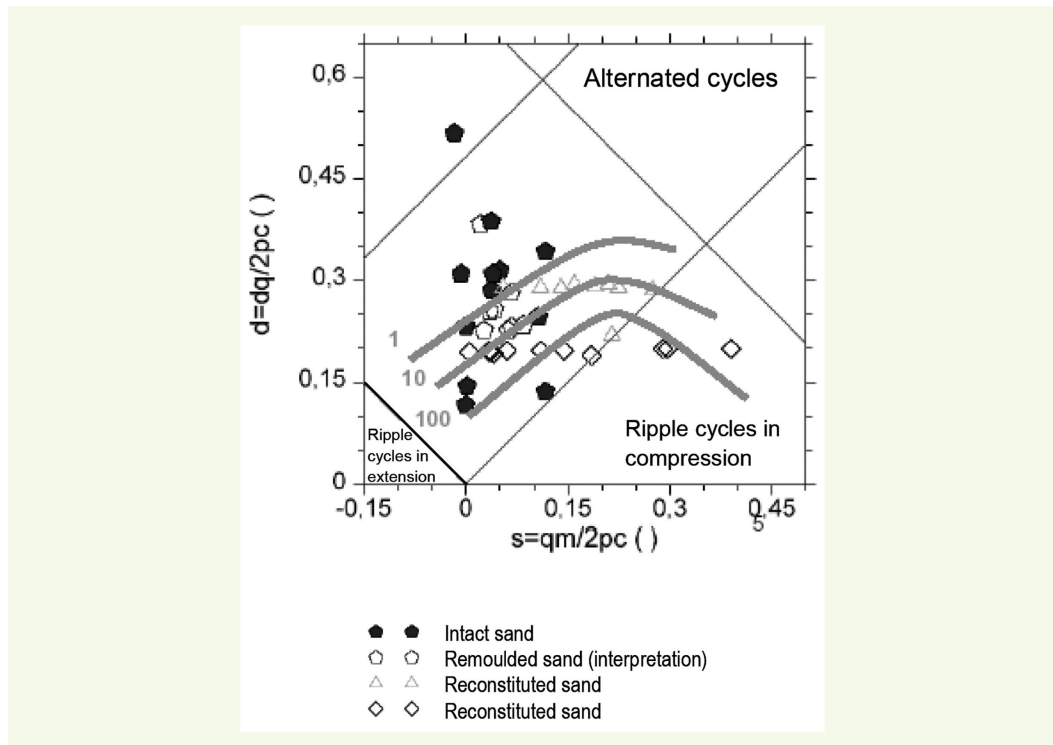
The Coulomb compression and tension lines drawn in planes ( $p, q$ ), as shown in **Figure 2**, were determined by means of both triaxial CU tests at monotonic loading and cyclic tests. Their slopes equal respectively  $M_c = 1.42$  and  $M_e = -0.96$ , corresponding to an angle of friction  $\phi' = 35$  degrees.

**Figure 3** displays the numbers of cycles required to satisfy the  $\varepsilon_{ay SA} = 2.5\%$  criterion with respect to the ratio  $q_{ave}/2p_c$  in the two series devoted to the reconstituted sand. Test results obtained on the intact sand have also been reported. The curves plotted show that with the reconstituted sand (triangles and diamonds), the number of cycles is maximized for an average deviator, such that  $q_{ave}/2p_c = 0.2$  approx.

**Figure 4** features the cyclic strength parameter expressed in the plane ( $q_{ave}/2p_c, \Delta q_y/2p_c$ ), where the cyclic amplitude  $\Delta q_y$  is presented as a function of the average deviator  $q_{ave}$  normalized by  $2p_c$ . In this particular plane, the two bisectors separate: the alternated tests at the centre, ripple tests in tension to the left, and ripple tests in compression to the right. These figures distinguish the intact sands from the recomposed sand (series 1 and 2). For the recomposed material, it is possible to depict the cyclic strength values obtained at 1, 10 and 100 cycles and the  $\varepsilon_{ay SA} = 2.5\%$  criterion. These curves also reach their maximum around  $q_{ave}/2p_c \approx 0.2$ .



**Figure 3**  
 Depiction of the number of cycles required to satisfy the  $\epsilon_{cy,SA} = 2.5\%$  criterion vs. the static stress ratio



**Figure 4**  
 Presentation of the cyclic amplitude required to satisfy the  $\epsilon_{cy,SA} = 2.5\%$  criterion vs. the static stress ratio

On average, the intact sands are more resistant, yet with dispersed levels of strength; they were not tested under highly asymmetric deviators. This type of parameterization cannot therefore be undertaken for such materials.

Two other lines with slopes of -1 and +1 were drawn on the plane  $(q_{ave}/2p_c, \Delta q_y/2p_c)$  in **Figure 4**; these lines represent sand strength. Beginning with the Coulomb equation in compression expressed on the effective stress plane  $(p, q)$  in the form  $q = M_c p$  (assuming the absence of cohesion) and then with  $q = q_{ave} + \Delta q_y$  and  $p = p_c$ , the following result is obtained:

$$\Delta q_y / 2p_c = +M_c/2 - q_{ave}/2p_c$$

Similarly, in tension, with  $q = q_{ave} - \Delta q_y$ , the following is derived:

$$\Delta q_y / 2p_c = -M_c/2 + q_{ave}/2p_c$$

In the case of the sand tested herein,  $\phi' = 35$  degrees,  $M_c = 1.42$  and  $M_c = -0.96$ , hence  $M_c/2 = 0.71$  and  $M_c/2 = -0.48$ . These lines seem to indicate a framing of the curves  $N_y = 1, 10$  and  $100$  plotted for the criterion at 2.5%.

The curves in **Figure 4**, which yield the cyclic strength at  $N_y$  cycles and presented as a function of cyclic loading dissymmetry, are comparable to those proposed by Vaid and Chern (1983, 1985), Mohamad and Dobry (1986), Hyodo *et al.* (1994) and Hyde *et al.* (2006) on materials reconstituted in the laboratory. According to these authors, the strength of sands features a maximum  $\Delta q_y/2p_c$  for a static level  $q_{ave}/2p_c$  that depends on sand density.

Loose sands are collapsible and display limited compressive strength and tensile strength, which explains why the maximum  $\Delta q_y/2p_c$  is located at levels below  $q_{ave}/2p_c$ , near a value of 0.1. In contrast, dense sands experience continuous strength increases for  $q_{ave}/2p_c$  ratio values exceeding 0.2. In all cases, the tensile strength values are small, which explains why the strain criteria, e.g.  $\epsilon_{ay SA} = 2.5\%$ , are reached more readily in tension during the strictly alternating campaign of cyclic triaxial tests.

## CONCLUSION

Three series of cyclic triaxial tests have been presented in this study. The first series focused on a clayey silt tested in its natural state and subjected to various cyclic loading rates. These tests demonstrate that the pore pressure response depends on loading rate. The quality of effective stress test interpretation thus depends on the choice of this rate. The cyclic frequencies typically introduced on clean sands exceed, and sometimes by a wide margin, the rates specified in triaxial testing procedures for monotonic loading tests on silty or clayey soils. The sample clayey silt tested herein suggests that these rates are still too fast to carry out the undrained cyclic loadings. The rate choice provided in the procedures for triaxial tests with monotonic loading does not therefore appear to be conservative.

The other two series of cyclic triaxial tests were conducted on a slightly silty, reconstituted sand sample. Each series was performed with a given cyclic amplitude, yet at different static stress levels. The cyclic strength of the reconstituted sand hits a maximum for an average shear level found in compression. Consequently, tensile strength values are low; as such, the strength criteria defined in strain are more readily reached in tension during the alternating cyclic triaxial tests.

## REFERENCES

- AFNOR** (Association Française de Normalisation, 1994). Sols : reconnaissances et essais. Essais à l'appareil triaxial de révolution. NF P 94-074, Octobre 1994, 36 p.
- BISHOP A.W., HENKEL D.J.** (1964). The measurement of soil properties in the triaxial test. Edward Arnold Publishers LTD, London, 228 p.
- GIBSON R.E., HENKEL D.J.** (1954). Influence of duration of tests at constant rate of strain on measured drained strength. *Géotechnique*, vol. 4, n° 1, pp. 6-15.
- HYDE A.F.L., HIGUCHI T., YASUHARA K.** (2006). Liquefaction, cyclic mobility and failure of silt. *J. Geotech. and Geoenviron. Engrg.*, vol. 132, n°6, pp. 716-735.
- HYODO M., TANIMIZU H., YASUFUKU N., T. MURATA H.** (1994). Undrained cyclic and monotonic triaxial behaviour of saturated loose sand. *Soils and Foundations*, vol. 34, n°1, pp. 19-32.
- MOHAMAD R., DOBRY R.** (1986). Undrained monotonic and cyclic triaxial strength of sand. *J. Soil Mech. and Found. Div., ASCE*, vol. 112, n°10, pp. 941-958.
- VAID Y.P., CHERN J.C.** (1983). Effect of static shear on resistance to liquefaction. *Soils and Foundations*, vol. 23, n°1, pp. 47-60. *Soils and Foundations*, vol. 25, n° 3, pp. 154-156.